PHYS 102 – General Physics II

Final Exam, May 29, 2008

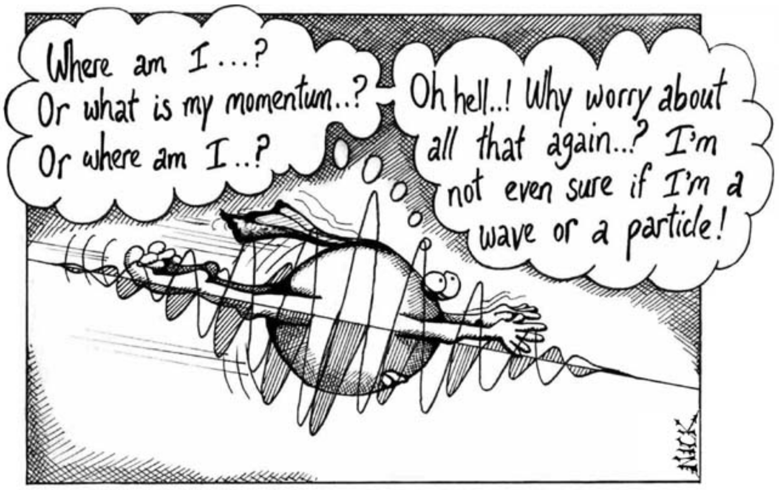
<u>Duration: 130 minutes</u>

NAME:... Section:......................

Q.1 (25)	Q.2 (25)	Q.3 (25)	Q.4 (25)	Total (100)

Suggestions:

1. Read the questions carefully.
2. State the solutions clearly and with necessary comments (explanations).
3. Write legibly.
4. Check your results in terms of dimensions, units, and special limits of the problem.



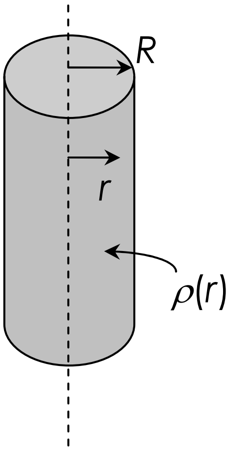
Photon self-identity problems.

Note: Solutions will be available just after the exam at

http://www.fen.bilkent.edu.tr/~phys102

Q.1 ***(25 points)*** **Electrostatics**

An infinitely-long nonconducting cylinder of radius R carries a nonuniform volume charge density $\rho(r) = \rho_0 \frac{r}{R}$, where ρ_0 is a known constant and r is the cylindrical radial distance from its axis (see the figure).



a) Using Gauss' law determine the electric field vector everywhere (i.e., $r<R$ and $r>R$),

b) Determine the corresponding electric potential everywhere, choosing $V(a)=0$, i.e., potential reference chosen at $r=a$ where $a>R$.

c) Can you choose infinity as potential reference? Give your reasoning.

Q.2 ***(25 points)*** **Magnetic Field**

Consider an infinite cylindrical wire of radius a centered along the z-axis with current density $\vec{J} = \hat{z}\, J_0$, where J_0 is a constant which is known.

(a) Draw the direction of the magnetic field vector inside and outside of the wire. (No calculation is required for this part.)

(b) Determine the total current that passes through this wire.

(c) What is the strength of the magnetic field inside the wire (as a function of the radial distance r from the wire axis)?

(d) What is the strength of the magnetic field outside the wire (as a function of the radial distance r from the wire axis)?

Q.3 ***(25 points)*** **RL Circuits**

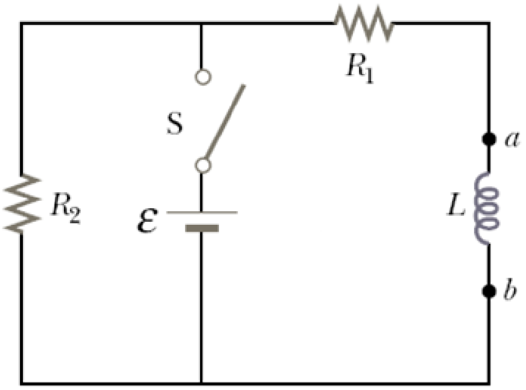
In the figure, the switch is kept closed for $t < 0$, and steady-state conditions are established. At $t = 0$ the switch is opened.

(a) Find the initial voltage V_L across the inductor L just after $t = 0$.

(b) Which end of the coil is at the higher potential: a or b?

(c) Plot the currents in R_1 and in R_2 as a function of time, treating the steady-state directions as positive. Show values before and after $t = 0$.

(d) How long after t =0 does the current in R_2 have the1/5 of its initial value?



Q.4 ***(25 points)*** **EM Waves**

A electromagnetic wave travelling in free-space has the electric field given by

$$\vec{E} = \hat{i}\ E_0 \sin\left(\omega t + \frac{\omega z}{c}\right) + 2\hat{j}\ E_0 \sin\left(\omega t + \frac{\omega z}{c}\right),$$

here, E_0 is a known constant.

(a) What is the direction of propagation of the wave? Show that $\omega/c = k$ is the wave number.

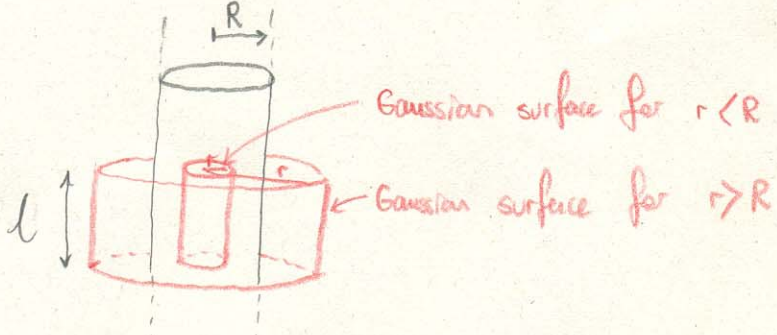
(b) What angle does the direction of polarization make with x, y and z axes?

(c) Write down the expression for the magnetic field vector of this wave as a function of space and time.

(d) What is the magnitude and direction of the Poynting vector?

Phys-102 Final Exam Solutions

1)

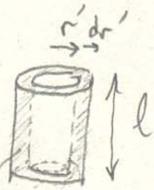


Due to cylindrical symmetry, the $\vec{E}$ field lines are cylindrically out

Therefore E-field flux will only be through lateral sides.

For $r < R$:

$$\underbrace{\varepsilon_0 \int_{S_{lateral}} \vec{E}\cdot\hat{n}\,da}_{} = Q_{enc} = \int_0^r \underset{\frac{\rho_0 r'}{R}}{\rho(r')}\, \underset{2\pi r' l\, dr'}{dv}$$



$$\varepsilon_0 E\, \cancel{2\pi} r \cancel{l} = \cancel{2\pi l}\,\frac{\rho_0}{R}\,\frac{r^3}{3}\,r^2$$

$$\vec{E}(r<R) = \frac{\rho_0}{3\varepsilon_0 R}\,r^2\,\hat{r} \leftarrow \text{cylindrical radial dir.}$$

For $r > R$:

$$\underbrace{\varepsilon_0 \int_{S_{lateral}} \vec{E}\cdot\hat{n}\,da}_{\varepsilon_0 E\, 2\pi r l} = Q_{enc} = \underbrace{\int_0^R \rho(r')\,dv}_{2\pi l\,\frac{\rho_0}{R}\,\frac{R^3}{3}}$$

$$\vec{E}(r>R) = \frac{\rho_0 R^2}{3\varepsilon_0 r}\,\hat{r}$$

$$V(r) - \underset{0}{\cancel{V(r=a)}} = -\int_a^r \vec{E}\cdot d\vec{l}$$

r>R r<R

For $r > R$: $$V(r) = -\int_a^r \frac{\rho_0 R^2}{3\varepsilon_0 r'}\,dr' = -\frac{\rho_0 R^2}{3\varepsilon_0}\ln\left(\frac{r}{a}\right)$$

For $r < R$: $$V(r) - \underbrace{V(R)}_{-\frac{\rho_0 R^2}{3\varepsilon_0}\ln(R/a)} = \underbrace{-\int_R^r \frac{\rho_0}{3\varepsilon_0 R}\,r'^2\,dr'}_{\frac{\rho_0}{3\varepsilon_0 R}\left(\frac{R^3}{3}-\frac{r^3}{3}\right)} \Rightarrow V(r<R) = \frac{\rho R^2}{3\varepsilon_0}\left[-\ln\left(\frac{R}{a}\right)+\frac{1}{3}-\frac{r^3}{3R^3}\right]$$

NB: Infinity cannot be used for potential reference, as the charge distribution extends to ∞.
You can also see this by the problems caused if we set a → ∞.

2]

a)

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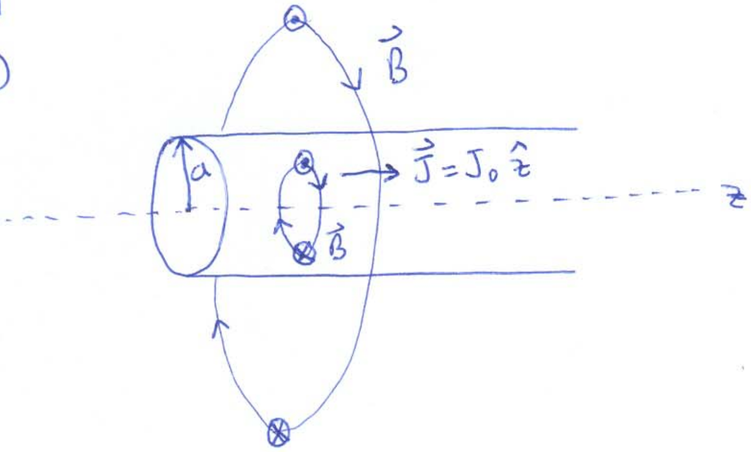


b) $I = \int \vec{J}\cdot d\vec{s} = \int J_0 \, ds = J_0 \int ds = J_0 \pi a^2$

$d\vec{s} = ds \, \hat{z}$

7//

c) $r < a$, from Ampere's law

7//

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I_{in} \longrightarrow B(2\pi r) = \mu_0 J_0 (\pi r^2) \longrightarrow B(r) = \frac{\mu_0 J_0}{2} r \, , \; r \le a$$

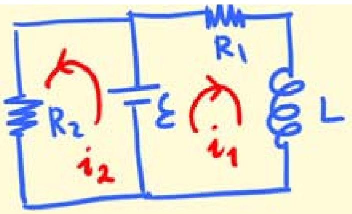
d) $r > a$, $\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I_{in} \longrightarrow B(2\pi r) = \mu_0 I = \mu_0 J_0 \pi a^2$

7//

$$B(r) = \frac{\mu_0 J_0 a^2}{2r} = \frac{\mu_0 I}{2\pi r} \, , \; r \ge a$$

3)

$t<0$



$$i_1(t) = \frac{\varepsilon}{R_1}\left(1 - e^{-t/\tau}\right)$$

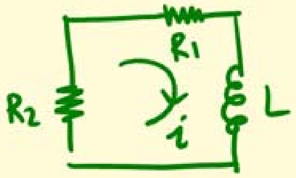
$$\tau = \frac{L}{R_1}$$

in steady-state

$i_1 = \varepsilon / R_1$ (clockwise)

$i_2 = \varepsilon / R_2$ (c. clockwise)

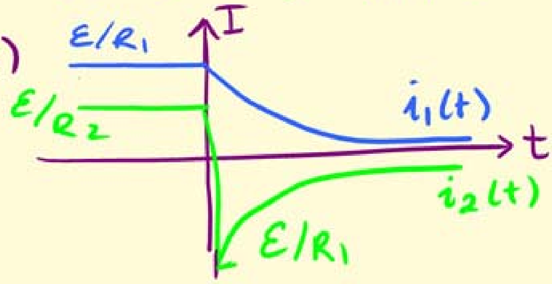
$t \geq 0$



$$i(t) = \frac{\varepsilon}{R_1} e^{-t/\tau}, \quad \tau = L/R_1+R_2$$

a) $V_L = (R_1+R_2)\, \varepsilon / R_1$, b) b is at higher potential

c)



d) $i(t) = \frac{\varepsilon}{R_1} e^{-t/\tau}$

$$\frac{\varepsilon}{5R_1} = \frac{\varepsilon}{R_1} e^{-t^*/\tau}$$

$$t^* = L/R_1+R_2 \ln(5)$$

4] An electromagnetic wave has the form

$$\vec{E} = \hat{x}E_0 \sin(\omega t + \frac{\omega z}{c}) + 2\,\hat{y}E_0 \sin(\omega t + \frac{\omega z}{c})$$

(a) What is the direction of propagation of the wave? Show that ω/c=k wave vector.

(b) What angle does the direction of polarization make with x, y, and z axes?

↳ This part will be graded as a BONUS!

(c) Write down a formula for the magnetic field of this wave as a function of space and time.

(d) What is the magnitude and direction of the Poynting vector?

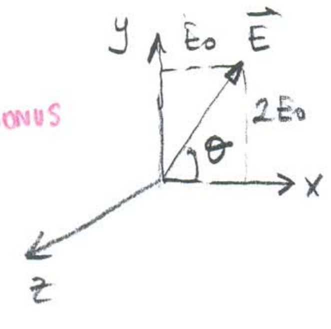
SOLUTION

(a) Waves having functonal forms $f(z+wt)$ propagate in $-\hat{z}$ direction.

$\omega = 2\pi\nu$ $\qquad \frac{2\pi\nu}{c} = \frac{2\pi}{\frac{c}{\nu}} = \frac{2\pi}{\lambda} = k$

(b)

↑BONUS



$\text{tn}\,\theta = \left(\frac{2E_0}{E_0}\right) \Rightarrow \theta = \text{tn}^{-1}(2)$

$\theta = 63°$ in the (xy)-plane

$\vec{E} + \hat{z}$

c) $B = \frac{E_0}{c} \Rightarrow \vec{B} = \hat{y}\frac{E_0}{c}\sin(wt + \frac{wz}{c}) + 2\hat{x}\,\frac{E_0}{c}\sin(wt + \frac{wz}{c})$

d) $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$: $\qquad \vec{S} = -\frac{3E_0^2}{\mu_0 c}\sin^2(wt + \frac{wz}{c})\,\hat{z}$

$|\vec{S}| = \frac{3E_0^2}{\mu_0 c}\sin^2(wt + \frac{wz}{c})$