PHYS 102 – General Physics II

Midterm Exam I, March 22, 2008

Duration: 100 minutes

NAME:.. Section:......................

Q.1 (35)	Q.2 (35)	Q.3 (35)	Total (105)

Suggestions:

1. Read the questions carefully.
2. State the solutions clearly and with necessary comments (explanations).
3. Write legibly.
4. Check your results in terms of dimensions, units, and special limits of the problem.

Note: Solutions will be available just after the exam at

http://www.fen.bilkent.edu.tr/~phys102

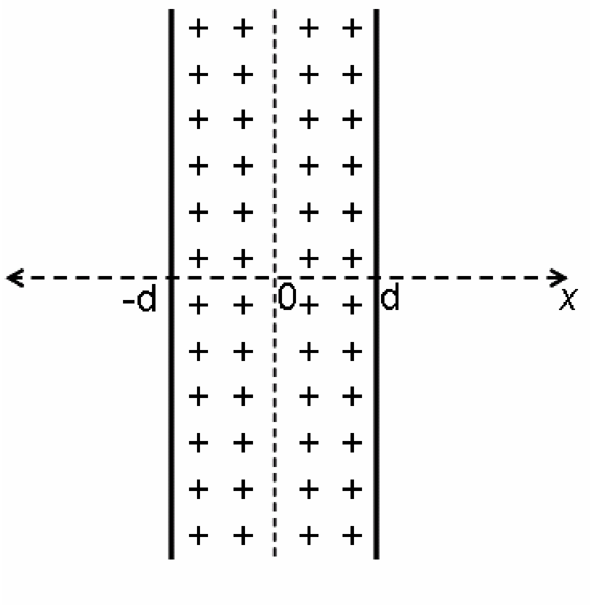
Q.1 ***(35 points)*** **Gauss' Law and nonconductors**

Consider an infinitely large slab (thick plane) with a thickness of $2d$, a part of which is shown in cross-section along x-axis below. The slab extends to infinity in y and z directions perpendicular to x direction. The slab is uniformly charged with a volume charge density $\rho>0$.

a) *(10 points)* What is the **electric field** right in the middle of the slab, i.e., at $x=0$?

b) *(15 points)* Find the **electric field vector** as a function of x inside the slab, i.e., for $|x|<d$.

c) *(10 points)* Find the **electric field vector** as a function of x outside the slab, i.e., for $|x|>d$.



Q.2 ***(35 points)*** **Electric Potential**

An insulating solid sphere of radius *R* has a uniform positive charge *Q* as shown in the Figure below.

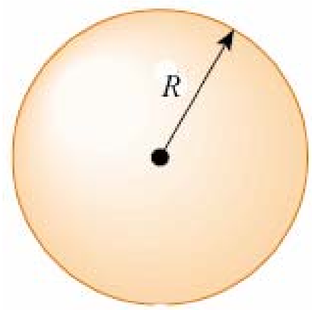
a) *(5 points)* Find charge density ρ.

b) *(10 points)* Find the electric potential at a point *r* outside the sphere (*r*>*R*). Take the potential to be zero at $r=\infty$.

c) *(10 points)* Find the potential at a point *r* inside the sphere (*r*<*R*).

d) *(5 points)* Plot potential *V(r)* as a function of the distance from the center.

e) *(5 points)* What can you say about the results (a), (b), and (c) if we have conducting sphere having the same amount of charge on it?



Q.3 ***(35 points)*** **Capacitance**

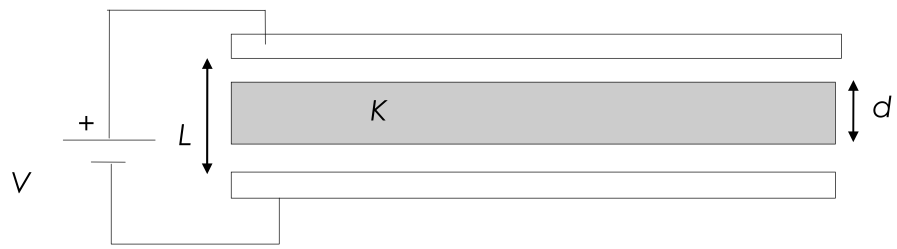
A dielectric slab of thickness d and dielectric constant K is inserted inside a capacitor with area A and thickness L. A voltage difference V is applied across the plates.

a) *(10 points)* Find the electric fields inside the air gap and inside the dielectric.

b) *(10 points)* Find the surface charge density on each of the plates.

c) *(5 points)* Use the information in (b) to obtain the capacitance.

d) *(10 points)* Find the capacitance of the system using capacitors connected in series, and compare your result with part (c).



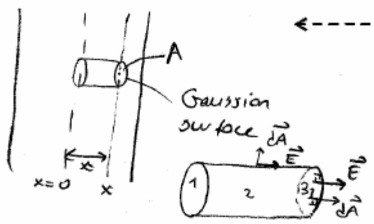
PHYS 102 First Midterm Exam Solutions March 22, 2008

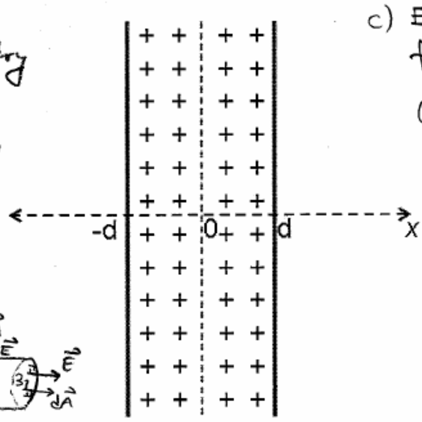
Q1 ***(35 points)*** **Gauss' Law and nonconductors**

a) $E = 0$ at $x = 0$ b/c planar symmetry

b) Gauss' Law 1 pt

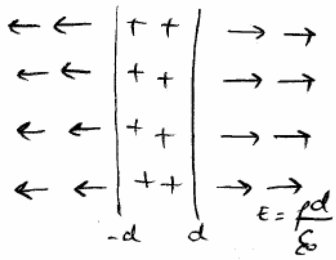
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enclosed}$$





c) E has to be constant for infinitely large plane.

@ $x = d \Rightarrow E = \frac{\rho d}{\varepsilon_0}$



LHS: $\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \int_1 \vec{E} \cdot d\vec{A} + \varepsilon_0 \int_2 \vec{E} \cdot d\vec{A} + \varepsilon_0 \int_3 \vec{E} \cdot d\vec{A} = \varepsilon_0 E . A$

On surface 1, $E = 0 \Rightarrow \varepsilon_0 \int \vec{E} \cdot d\vec{A} = 0$

On surface 2, $\theta = 90° \Rightarrow \varepsilon_0 \int \vec{E} \cdot d\vec{A} = 0$

On surface 3, $\theta = 0° \Rightarrow \varepsilon_0 \int \vec{E} \cdot d\vec{A} = \varepsilon_0 . E . A$

E: constant (due to planar symmetry)

RHS: $q_{enclosed} = \rho . Ax$

LHS=RHS: $\varepsilon_0 E . A = \rho . A . x$

$E(x) = \frac{\rho}{\varepsilon_0} . x$ in $\hat{x}$ direction. 1pt

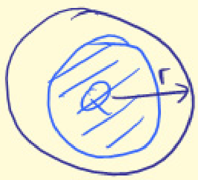
$\vec{E}(x) = \frac{\rho}{\varepsilon_0} x . \hat{x} = \frac{\rho}{\varepsilon_0} \cdot \vec{x}$

Q2 ***(35 points)*** **Potential**

Solution:

a.) charge density $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$

b) $V(r>R) = ?$



Electric field at point r, from Gauss law

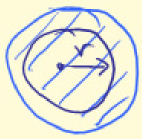
$$\oint \vec{E}\cdot d\vec{A} = Q/\epsilon_0 \Rightarrow E(r) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}$$

potential at point r:

$$V_r - V_\infty = -\int_\infty^r \vec{E}(r)\cdot d\vec{r} = -\int_\infty^r \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}dr$$

$$V(r>R) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}, \quad V(r=R) = \frac{Q}{4\pi\epsilon_0}\frac{1}{R}$$

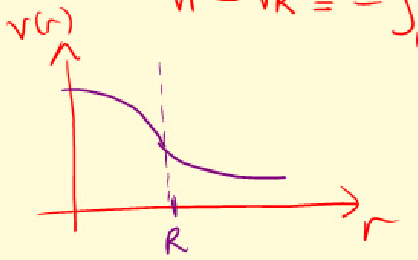
c) $V(r<R) = ?$ Electric field inside the sphere, from Gauss law



$$\oint \vec{E}\cdot d\vec{A} = \rho\cdot\frac{4}{3}\pi r^3/\epsilon_0 \Rightarrow E(r) = \frac{Q}{4\pi\epsilon_0 R^3}\, r$$

$$V_r - V_R = -\int_R^r E(r)\,dr \Rightarrow V(r<R) = \frac{Q}{8\pi\epsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)$$

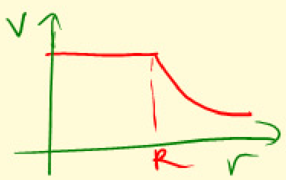
d)



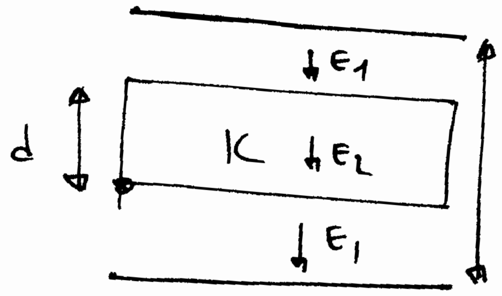
e) a) charge density $\rho = 0$ inside conductor

b) $V(r>R) = \dfrac{Q}{4\pi\epsilon_0}\dfrac{1}{r}$

c) $V(r<R) = \dfrac{Q}{4\pi\epsilon_0}\dfrac{1}{R}$



Q3: Solution (A)



E_1 : E-FIELD IN AIR

E_2 : " IN DIELECTRIC

$$E_2 = \frac{E_1}{K} \quad (1)$$

$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{\ell} \Rightarrow \Delta V = E_2 \cdot d + E_1 (L-d) \quad (2)$$

Use (1) in (2)

$$\Delta V = V = \frac{E_1}{K}\cdot d + E_1 (L-d) = E_1 \left(\frac{d}{K} + (L-d)\right)$$

$$\boxed{E_1 = \frac{V}{\frac{d}{K} + (L-d)}} \quad (3)$$

Use (3) in (1)

$$\boxed{E_2 = \frac{V}{d + K(L-d)}} \quad (4)$$

(B)

$$E_1 = \frac{\sigma}{\epsilon_0} \quad (5) \qquad \text{Use (3) in (5)}$$

$$\boxed{\sigma = \epsilon_0 \cdot \frac{V}{\frac{d}{K} + (L-d)}} \quad (6)$$

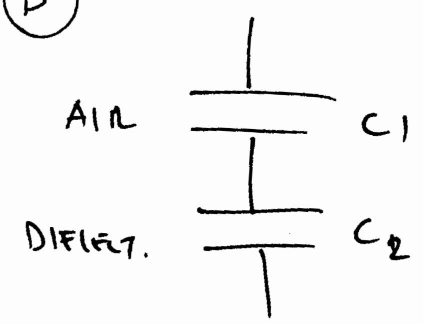
(C)

$$Q = \sigma \cdot A = \frac{\epsilon_0 \cdot V \cdot A}{\frac{d}{K} + (L-d)}$$

$$Q = CV \Rightarrow C = \frac{Q}{V}$$

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon_0 \cdot A}{\frac{d}{K} + (L-d)}} \qquad (7)$$

(D)



$$C_1 = \frac{\epsilon_0 \cdot A}{L-d}$$

$$C_2 = \frac{K \cdot \epsilon_0 \cdot A}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\epsilon_0 \cdot A} \cdot \left(\frac{d}{K} + (L-d) \right)$$

$$\Rightarrow \boxed{C_{eq} = \frac{\epsilon_0 \cdot A}{\frac{d}{K} + (L-d)}} \qquad (8)$$