PHYS 102 – General Physics II

Midterm Exam II, April 26, 2008

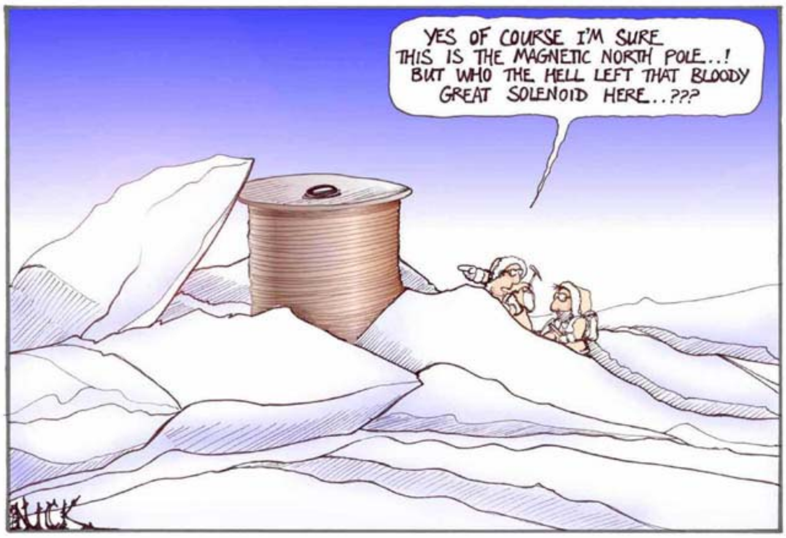
Duration: 100 minutes

NAME:.. Section:......................

Q.1 (35)	Q.2 (35)	Q.3 (35)	Total (105)

Suggestions:

1. Read the questions carefully.
2. State the solutions clearly and with necessary comments (explanations).
3. Write legibly.
4. Check your results in terms of dimensions, units, and special limits of the problem.

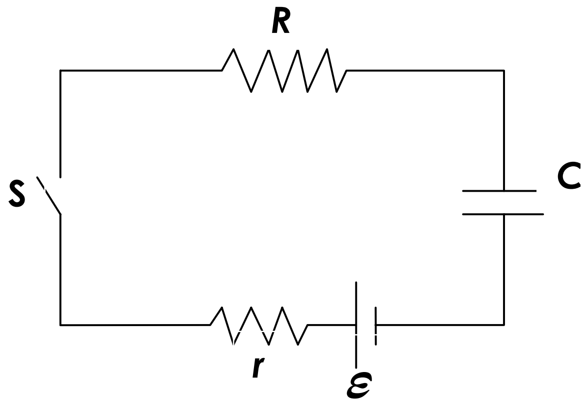


Note: Solutions will be available just after the exam at

http://www.fen.bilkent.edu.tr/~phys102

Q.1 ***(35 points)*** **RC Circuit**

An EMF source with potential difference $\boldsymbol{\mathcal{E}}$ and internal resistance $\boldsymbol{r}$ is connected to a resistor $\boldsymbol{R}$ and a capacitor with capacitance $\boldsymbol{C}$ in series connection through a switch **S** as in the figure. Initially there is no charge in the capacitor and at t=0 switch **S** is closed. Write your answers in terms of $\boldsymbol{\mathcal{E}}$, $\boldsymbol{r}$, $\boldsymbol{R}$, $\boldsymbol{C}$ and $\boldsymbol{t}$.



(a) (5pts) Show the direction of the positive current flow and write down the equation for charge on the capacitor for t>0, using Kirchhoff's rules. (Note that your equation should include charge q, only (not current) as a dependent variable.

(b) (5pts) Show that $q(t) = C\varepsilon\left(1-e^{-t/\tau}\right)$, is the solution for the equation you write in part (a) by directly substituting it into equation provided that τ is chosen properly. Determine τ (also specify its SI unit).

(c) (5pts) What is the initial current at t=0 when switch **S** is closed? What is the current passing through the circuit at t>0?

(d) (5pts) What is the maximum amount of charge that can be stored in the capacitor and when is this amount of charge accumulated in the capacitor?

(e) (5pts) Plot current versus time and charge versus time graphs. Show the initial and final current/charge values in the graphs.

(f) (5pts) How much energy is stored in the capacitor at some time t_1>0?

(g) (5pts) How much heat (energy) has been generated in the resistor R from t=0 to $t=t_1$?

Q.2 ***(35 points)*** **Magnetic Field**

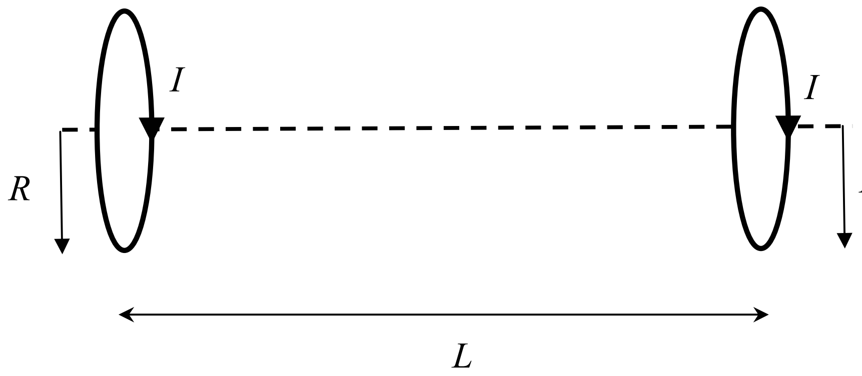
Two current loops with equal current I (both oriented in the same direction) and radii R are separated by a long distance L as shown in the figure. L is much larger than R.

(a) Find the magnetic moments of both current loops.

(b) Find the magnetic field generated by the loop on the left at the center of the other loop on the right

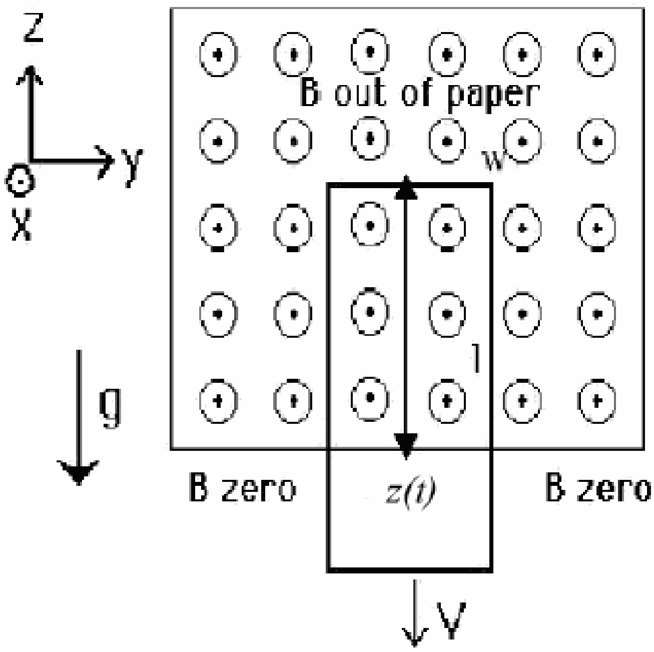
(c) Show that, in order to separate these loops infinitely far apart, one has to do a positive work given by

$$W = \mu_0 \frac{\vec{\mu}_1 \cdot \vec{\mu}_2}{2\pi \left(L^2 + R^2 \right)^{3/2}}.$$



Q.3 ***(35 points)*** **Electromagnetic Induction**

A rectangular loop of wire with mass m, width w, vertical length l, and resistance R falls out of a magnetic field under the influence of gravity. The magnetic field is uniform and out of the paper ($\vec{B} = B\hat{x}$) within the area shown (see sketch) and zero outside of that area. At the time shown in the sketch, the loop is exiting the magnetic field at speed $\vec{V}(t) = V(t)\hat{z}$, where $V(t) < 0$ (meaning the loop is moving downward, not upward). Suppose at time t the distance from the top of the loop to the point where the magnetic field goes to zero is $z(t)$ (see sketch).



(a) What is the relationship between $V(t)$ and $z(t)$? Be careful of your signs here, remember that $z(t)$ is positive and decreasing with time, so $dz(t)/dt < 0$.

(b) If we define the area vector $\vec{A}$ to be out of the page, what is the magnetic flux ϕ_B through our circuit at time t (in terms of $z(t)$, not $V(t)$).

(c) What is $d\phi_B / dt$? Is this positive or negative at time t? Be careful here, your answer should include $V(t)$ (not $z(t)$ and remember that $V(t) < 0$).

(d) What is the direction (clockwise or counterclockwise) and magnitude of the induced current on the loop of wire?

(e) What is the direction (into the page or out of the page) of the self-magnetic field due to the induced current inside the circuit loop?

(f) Besides gravity, what other force acts on the loop in the ± **z**-direction? Give the magnitude and direction of this force in terms of the quantities given. (Hint: use $d\vec{F} = Id\vec{l} \times \vec{B}$)

(g) What is the magnitude of the terminal velocity?

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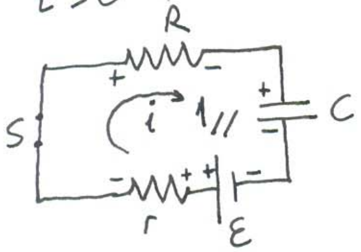
1) 35pts

a) 5pts

$t>0$



1// $\varepsilon - ir - iR - \frac{q}{C} = 0$

1// $i = \frac{dq}{dt}$

2// $\varepsilon - (r+R)\frac{dq}{dt} - \frac{q}{C} = 0$

b) 5pts $q(t) = C\varepsilon(1-e^{-t/\tau})$

1// $\frac{dq}{dt} = \frac{C\varepsilon}{\tau} e^{-t/\tau}$

$\varepsilon - (r+R)\frac{C\varepsilon}{\tau} e^{-t/\tau} - \varepsilon(1-e^{-t/\tau}) = 0$

$\cancel{\varepsilon} - \frac{(r+R)C}{\tau}\varepsilon e^{-t/\tau} - \cancel{\varepsilon} + \varepsilon e^{-t/\tau} = 0$ 1//

So $q(t)$ is a solution iff $\frac{(r+R)C}{\tau} = 1 \rightarrow \underbrace{\tau = (r+R)C}_{2//}$, $\underbrace{\text{seconds}}_{1//}$

c) 5pts

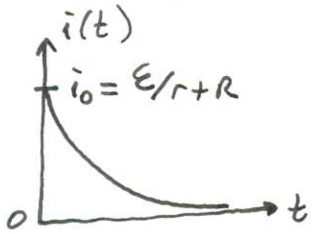
1// at $t=0 \rightarrow q(0)=0$, $\varepsilon - i_0(r+R) = 0 \rightarrow i_0 = \frac{\varepsilon}{r+R}$ 2//

for $t>0 \rightarrow i(t) = \frac{dq}{dt} = \frac{C\varepsilon}{\tau} e^{-t/\tau}\Big|_{\tau=(r+R)C} = i_0 e^{-t/(r+R)C}$ 2//

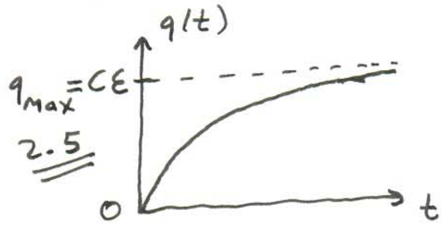
d) 5pts $q_{max} = q(\underbrace{t\rightarrow\infty}_{2//}) = \underbrace{C\varepsilon}_{3//}$ since $\lim_{t\to\infty} e^{-t/\tau} \rightarrow 0$

e) 5pts

2.5



2.5



f) 5pts $U_c(t_1) = \underbrace{\frac{q^2(t_1)}{2C}}_{2//} = \underbrace{\frac{1}{2}C\varepsilon^2(1-e^{-t_1/\tau})^2}_{3//}$ where $\tau = (r+R)C$

g) 5pts Power dissipation on resistor R ; $P(t) = i^2(t)R = i_0^2 R e^{-2t/(R+r)C}$ 2//

Energy dissipated from $t=0$ to $t=t_1$; $W = \int_0^{t_1} P(t)dt = i_0^2 R \int_0^{t_1} e^{-2t/(R+r)C} dt$

$W = i_0^2 \frac{R(R+r)C}{2}(1-e^{-2t_1/(R+r)C})$ 3//

where $i_0 = \varepsilon/(r+R)$

Q. 2 (35 pts.)

a) $\vec{\mu_1} = I\vec{A} = I\pi R^2 \vec{n}$

$\vec{\mu_1} = \vec{\mu_2}$

b) for $L \gg R$ the B-field at the center of the other loop generated by the magnetic dipole in the first loop is

$$\vec{B_1} = \mu_0 \frac{I R^2 \vec{n}}{2(L^2+R^2)^{3/2}} = \frac{\mu_0}{2\pi} \frac{\vec{\mu_1}}{(L^2+R^2)^{3/2}}$$

c) the total potential energy of this system is

$U = -\vec{\mu_2}\cdot\vec{B_1}$ or alternatively $U = -\vec{\mu_1}\cdot\vec{B_2}$

$$= -\frac{\mu_0}{2\pi} \frac{\vec{\mu_1}\cdot\vec{\mu_2}}{(L^2+R^2)^{3/2}} < 0 \quad \text{since } \vec{\mu_1} = \vec{\mu_2}.$$

$$= -\frac{\mu_0}{2\pi} \frac{|\mu_1|^2}{(L^2+R^2)^{3/2}}$$

to separate these loops one has to bring the loops infinitely apart by doing a work to bring the potential energy to zero hence the work to be done is;

$$W = \frac{\mu_0}{2\pi} \frac{|\mu_1|^2}{(L^2+R^2)^{3/2}} > 0$$

③ 35pts

a) 5pts

$$\left.\begin{array}{l}\vec{v}(t) = v(t)\,\hat{z}, \quad v(t) < 0 \\ z(t) > 0\end{array}\right\} \quad \frac{dz(t)}{dt} = v(t) < 0 \qquad 5//$$

b) 5pts

$$\phi_B = \int_S \vec{B}\cdot d\vec{a} = \int_S B\,da\cos 0 = B\int_S da = B\underbrace{w\,z(t)}_{\text{area of the loop where } B\neq 0} > 0 \qquad 4//$$

1// S: rectangular loop area

c) 5pts

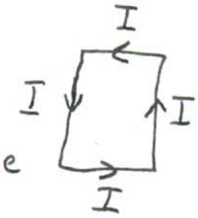
$$\frac{d\phi_B}{dt} = \frac{d}{dt}\left(Bw\,z(t)\right) = Bw\frac{dz(t)}{dt} = Bw\,v(t) < 0 \qquad 1//$$

4//

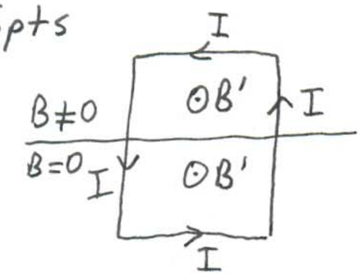
d) 5pts

2// $$\mathcal{E} = -\frac{d\phi_B}{dt} = -wB\,v(t) > 0$$

2// $$I = \frac{|\mathcal{E}|}{R} = \frac{wB\,|v(t)|}{R}$$, counterclockwise 1//

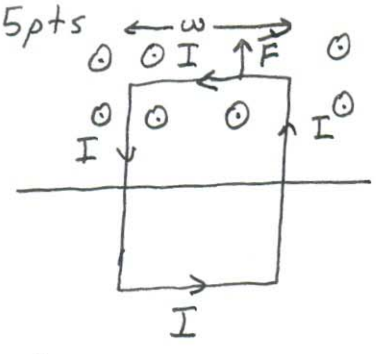


e) 5pts



B' is out of page from righthand rule due to induced current inside the loop.
5//

f) 5pts

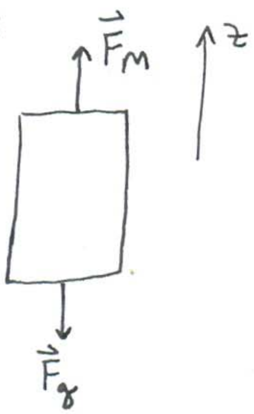


On the upper side of the rectangular loop there is force $\vec{F} = F\hat{z}$ due to induced current, I and magnetic field B ie,

$$\vec{F} = F\hat{z}, \quad \vec{F} = I\,\vec{L}\times\vec{B}, \quad F = IwB$$

2// 3//

g) 5pts



When $|\vec{F}_M| = |\vec{F}_g|$, $a = 0$ in z-direction ie v will be constant $v = v_{terminal}$

1// $$F_M = F_g$$

1// $$IwB = mg \quad \text{where } I = \frac{wB\,v_{terminal}}{R}$$

$$\frac{w^2B^2\,v_{ter}}{R} = mg \longrightarrow v_{ter} = \frac{Rmg}{w^2B^2} \qquad 3//$$