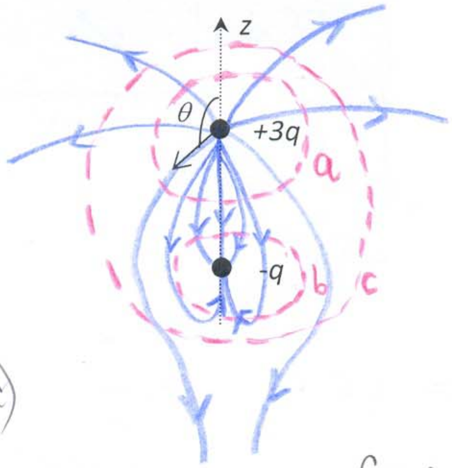
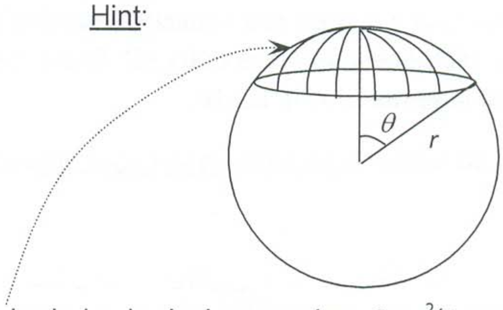
Q.1 *(25 points)* Electrostatics

Two charges $+3q$ and $-q$ are placed on the z-axis as shown. **a)** All the field lines start at $+3q$, however some end at $-q$ and others go to infinity. What fraction of the electric field lines starting from $+3q$ end at q? **b)** What is the maximum angle θ at which a field line starting from $+3q$ will end at infinity? (Specify just $\cos\theta$). *Provide full details and insert your numerical answers into boxes below.*



Hint:



Area of the shaded spherical cap region= $2\pi r^2(1-\cos\theta)$

a)

Gauss' Law

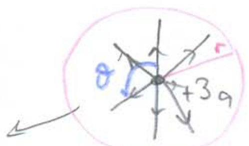
$$\Phi_a = \frac{3q}{\varepsilon_0} \leftarrow \text{Electric flux leaving } +3q$$

$$\Phi_b = \left|\frac{-q}{\varepsilon_0}\right| \leftarrow \text{" " entering to } -q$$

$$\Phi_c = \frac{3q-q}{\varepsilon_0} \leftarrow \text{" " going to } \infty$$

$\Rightarrow$ The fraction of electric field lines starting from $+3q$ and ending at $-q = \dfrac{\Phi_b}{\Phi_a} = \dfrac{1}{3}$ //

b) To track the critical ∡ θ, we should go almost on top of charge $+3q$. Here the total electric field is controlled by that of $+3q$ charge alone, which spreads uniformly in all directions.



distributed uniformly over area $4\pi r^2$ ← Total flux out $= \dfrac{3q}{\varepsilon_0}$

Flux to $\infty = \dfrac{2q}{\varepsilon_0}$

distributed uniformly over $2\pi r^2(1-\cos\theta)$

$$\frac{\Phi_c}{\Phi_a} = \frac{2}{3} = \frac{2\pi r^2(1-\cos\theta)}{4\pi r^2}$$

$$\Rightarrow \cos\theta = -1/3 \quad //$$

Put answers in these boxes

a) Fraction= 1/3

b) $\cos\theta$= $-1/3$

Q.2 ***(25 points)*** **Magnetic Force**

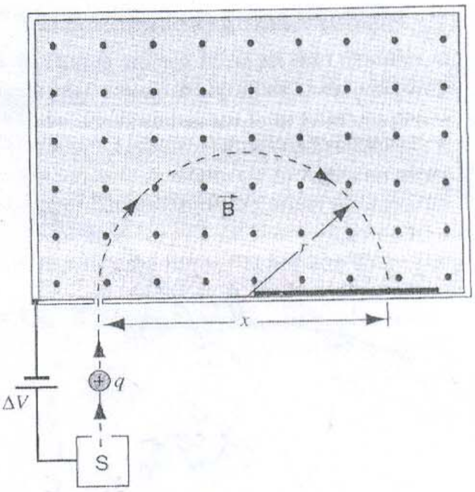


Figure on the right shows an arrangement used to measure the masses of ions. An ion of mass *m* and charge *+q* is produced esentially at rest in source S, a chamber in which gas discharge is taking place. The ion is acclereated by potential difference *ΔV* and allowed to enter a magnetic field **B**. In the field it moves in a semicircle striking a photographic plate at distance *x* from the entry slit. Determine the ion mass *m* in terms of *x, q,* B, *ΔV.*

Derive all relations you use, insert your answer in box below.

This is the cyclotron motion under the radial magnetic force: $F_M = q v B = m a$

$a = v^2/R$ uniform circular motion

$\Rightarrow \quad R = \dfrac{mv}{qB}$

$x = 2R = \dfrac{2mv}{qB}$ ← we have to determine this in terms of given quantities

A charge gains a kinetic energy $q\Delta V$ from a potential difference ΔV

$K = \frac{1}{2} m v^2 = q\,\Delta V \quad \Rightarrow \quad v = \sqrt{\dfrac{2q\,\Delta V}{m}}$

$\Rightarrow \quad x = \dfrac{2m}{qB}\sqrt{\dfrac{2q\Delta V}{m}} = \dfrac{2\sqrt{2}}{B}\sqrt{\dfrac{\Delta V m}{q}}$

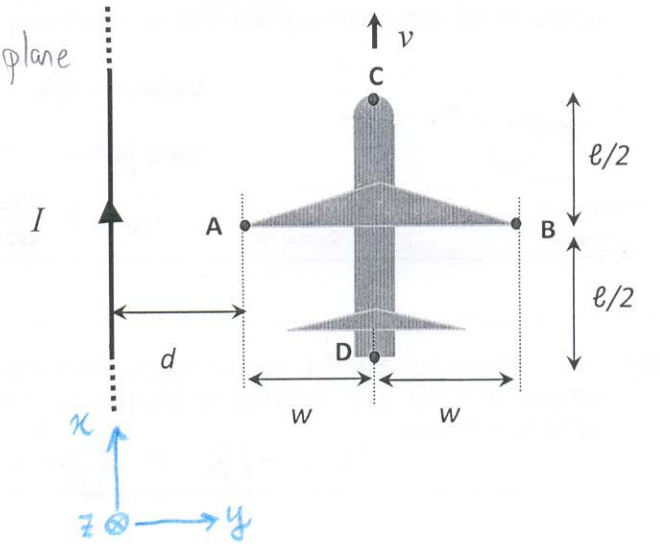
$\Rightarrow \quad m = \dfrac{B^2 q x^2}{8\Delta V}$

Put answer in this box

$m = \dfrac{B^2 q x^2}{8\Delta V}$

Q.3 *(25 points)* Induction

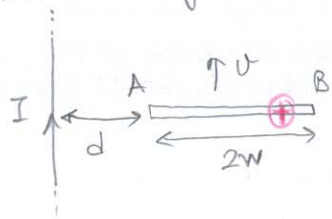
An airplane with a speed v is flying at the same altitude of a high voltage cable carrying a current I as shown in the figure. Determine the following potential differences: V_{BA}, V_{CD}, V_{CA}. (Notation: $V_{ij}=V_i-V_j$)



Since the plane flies on xy plane

$$\vec{B}(y) = \frac{\mu_0 I}{2\pi y}\hat{k}$$

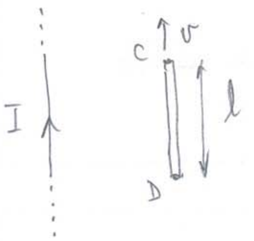
First consider just a conducting bar



The force on a +ve charge:

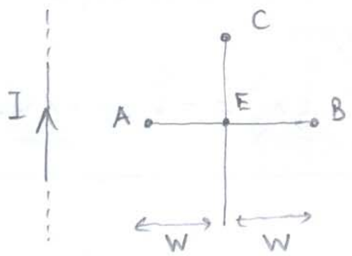
$$\vec{F}_q = q\vec{v}\times\vec{B} = qvB(-\hat{j})$$

$$V_{BA} = V_B - V_A = \int_A^B \vec{v}\times\vec{B}\cdot \vec{dl} \quad \leftarrow \hat{j}\,dy \quad \Rightarrow \quad V_B - V_A = \int_d^{d+2w} -v\frac{\mu_0 I}{2\pi y}dy \Rightarrow V_{BA} = -\frac{\mu_0 I v}{2\pi}\ln\left(\frac{d+2w}{d}\right)$$



$$\left.\begin{aligned} \vec{v}\times\vec{B} &= vB(-\hat{j}) \\ \vec{dl} &= \hat{i}\,dx \end{aligned}\right\} \quad \vec{v}\times\vec{B}\cdot\vec{dl} \equiv 0 \quad \Rightarrow V_{CD} = \cancel{\int_D^C \vec{v}\times\vec{B}\cdot\vec{dl}}^{\,0}$$

same reason



$$V_{CA} = \cancel{V_{CE}}^{\,0} + \underbrace{V_{EA}}_{-\frac{\mu_0 I v}{2\pi}\ln\left(\frac{d+w}{d}\right)} = -\frac{\mu_0 I v}{2\pi}\ln\left(\frac{d+w}{w}\right)$$

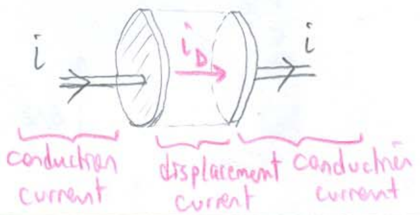
Put answers in these boxes

$V_{BA} = -\frac{\mu_0 I v}{2\pi}\ln\left(\frac{d+2w}{d}\right)$	$V_{CD} = 0$	$V_{CA} = -\frac{\mu_0 I v}{2\pi}\ln\left(\frac{d+w}{d}\right)$

Q.4 ***(25 points)*** **Conceptual Questions**

Answer the following questions **in** the box for each part. To claim any credit for your discussions, **base your reasoning on physics (laws).**

(a) (5 pt.) Consider the charging of a capacitor in an R-C circuit. How does the current pass from one plate to the other plate of the capacitor even though the region between the plates of the capacitor is filled with an insulating (i.e., non-conducting) material?



Between the plates, the conduction current is transferred to the deplacement current:

$$i_D = \varepsilon \frac{d\Phi_E}{dt}$$

(b) (5 pt.) Consider an R-L circuit made from the series connection of a batter of 10 V, a resistor of 1 kΩ, and inductor of 10 mH, what is the time constant of this circuit? (Give a numerical value)

$$\tau = L/R = \frac{10 \cdot 10^{-3}}{10^3} = 10\,\mu s$$

(c) (5 pt.) What is the SI unit for the product of electric and magnetic field magnitudes (i.e. *EB*)? (You must simplify your answer as much as possible)

$$\frac{V}{m} \times \frac{V}{m} \cdot \frac{s}{m} = \frac{V^2 s}{m^3} = kg^2 \frac{m}{C^2 s^3}$$

(d) (5 pt.) What does the Lenz' law state?

"The direction of any magnetic induction effect is such as to oppose the cause of the effect."

(e) (5 pt.) If the magnetic field vector of a plane wave is $\hat{j}B_0$ and its electric field is $\hat{k}E_0$, (both B_0, E_0 are positive) determine the direction of propagation of this wave?

The EM wave propagates along $\vec{E} \times \vec{B}$ direction: $\hat{k} \times \hat{j} = -\hat{i} \Rightarrow$ $\underline{-x \text{ dir.}}$