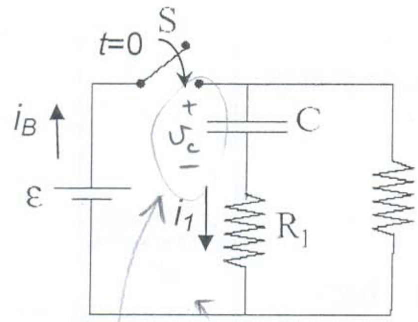
Q.1 *(25 points)* RC Circuit

In the circuit shown below the capacitor is <u>initially uncharged</u>. The switch S is closed at t=0. Determine <u>expressions</u> for the currents $i_B(t)$, $i_1(t)$, $i_2(t)$ and <u>plot</u> each one, indicating their values as $t \to 0$ and $t \to \infty$. *In your answers, use <u>only</u> the variables supplied with this problem.*



After the switch is closed:

$$i_B = i_1 + i_2 \quad \text{(junction rule)}$$

$$\mathcal{E} - i_2 R_2 = 0 \quad \text{(loop rule - outer loop)}$$

$$\boxed{i_2 = \frac{\mathcal{E}}{R_2}}$$

$$\mathcal{E} - \underset{\frac{q}{C}}{v_c} - \underset{\frac{dq}{dt}}{i_1} R_1 = 0 \quad \text{(loop rule)}$$

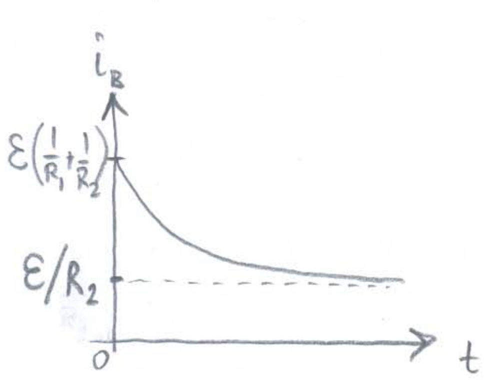
subject to $q(t=0) = 0$

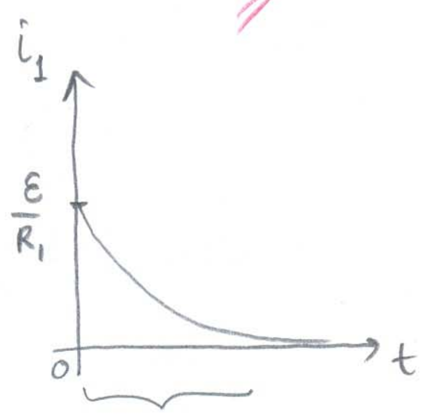
The solution for capacitor charge:

$$q(t) = C\mathcal{E}\left[1 - e^{-t/(R_1 C)}\right]$$

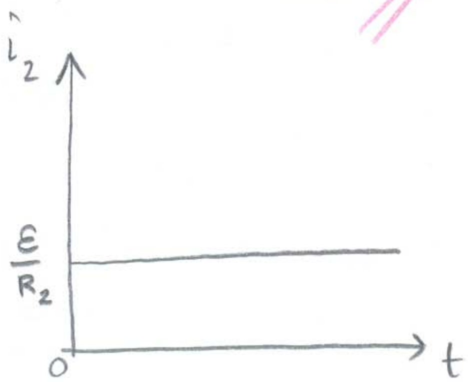
$$\boxed{i_1(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R_1} e^{-t/(R_1 C)}}$$

$$\boxed{i_B(t) = \frac{\mathcal{E}}{R_1} e^{-t/(R_1 C)} + \frac{\mathcal{E}}{R_2}}$$





decays in about $5\tau = 5R_1C$

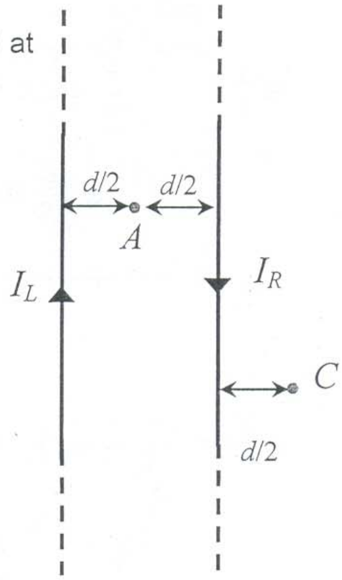


Q.2 ***(25 points)*** **Magnetic Field**

Two infinitely-long parallel wires separated by a distance d carry currents I_L and I_R in opposite directions as shown in the figure below. The current I_L is adjusted so that the total magnetic field at point C becomes zero.

(a) (10 pt.) Determine this current I_L in terms of I_R.

(b) (15 pt.) Find the magnitude and the direction of the magnetic field at point A (midway between the wires).



(a) The contributions of $\vec{B}$ field at point C of I_L and I_R are in opposite directions (RHR)

$$B(c) = |\vec{B}_L(c) - \vec{B}_R(c)|$$

$$\frac{\mu_0 I_L}{2\pi\left(\frac{3d}{2}\right)} - \frac{\mu_0 I_R}{2\pi \frac{d}{2}}$$

To have zero B field at point C

$$\boxed{I_L = 3 I_R}$$

(b) Using RHR, both contributions to point A are in the same direction

$$B(A) = \frac{\mu_0 \overbrace{I_L}^{3I_R}}{2\pi \frac{d}{2}} + \frac{\mu_0 I_R}{2\pi \frac{d}{2}}$$

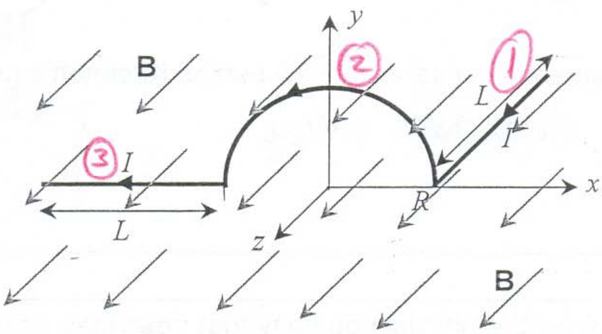
$$\Rightarrow \boxed{B(A) = \frac{4\mu_0 I_R}{\pi d}}$$

Direction of B(A) is into page $\otimes$

RHR

Q.3 *(25 points)* **Magnetic Force**

Consider the three sections of a wire carrying a current I: two straight sections of length L are aligned with the z and x-axes, and there is a semicircular part in between of radius R as shown in the figure. This is in a uniform magnetic field $\vec{B} = B\hat{k}$. Determine the total magnetic force vector on these sections (use the coordinate system given with the drawing).



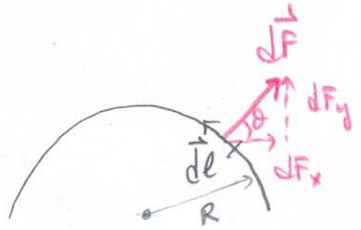
[This is example 27.8 from your textbook. Full solution is given on pages 934-35 which is reproduced here.]

Consider magnetic forces on each segment separately. $\vec{F}_m = \vec{F}_{M,1} + \vec{F}_{M,2} + \vec{F}_{M,3}$

Segment 1 : $\vec{F}_{M,1} \equiv 0$ since $\vec{L} \times \vec{B} = 0$

Segment 3 : $\vec{F}_{M,3} = ILB\,\hat{j}$

Segment 2 – semicircle :



$dF = I\,(R\,d\theta)\,B$

$dF_x = IR\,d\theta\,B\cos\theta$

$dF_y = IR\,d\theta\,B\sin\theta$

$$F_x = IRB\int_0^{\pi}\cos\theta\,d\theta = 0$$ (also expected from symmetry)

$$F_y = IRB\int_0^{\pi}\sin\theta\,d\theta = 2IRB$$

$\therefore$ Total Magnetic Force $\vec{F}_m = IB(L+2R)\,\hat{j}$

Q.4 ***(25 points)*** **Conceptual Questions**

Answer the following questions **in** the box for each part. To claim any credit for your discussions, **base your reasoning on physics (laws).**

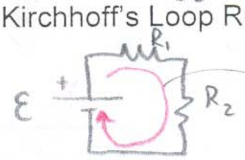
(a) (3 pt.) What is the fundamental conservation law behind Kirchhoff's Junction Rule?

Conservation of electric charge.

Charge at the junction is conserved

(3 pt.) What is the fundamental conservation law behind Kirchhoff's Loop Rule?

Conservation of electrostatic energy.



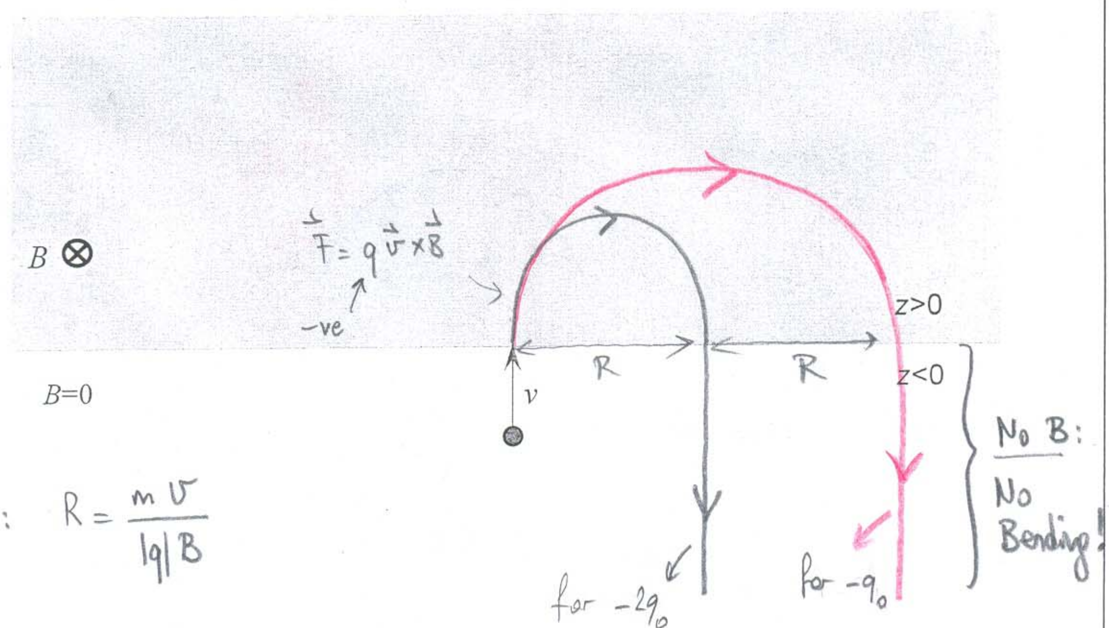
electrostatic energy is conserved around the loop.

(b) (4 pt.) What is the name for the physical quantity that describes how fast electrical energy dissipates across a resistor? What is the corresponding SI unit for this quantity?

Power

Watts (W)

(c) (6 pt.) A negatively charged particle -q_0 enters with a velocity $\vec{v} = \hat{k}v$ to uniform magnetic field zone (z>0) $\vec{B} = \hat{j}B$, which is directed into page as shown below. i) Sketch the full trajectory of the particle, ii) On the same drawing, show the trajectory for the case when the charge is doubled to -2q_0. *Cleary label which one is which.*



Use: $R = \dfrac{mv}{|q|B}$

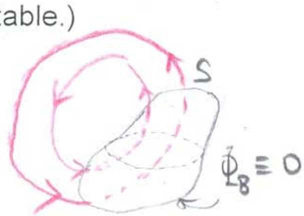
(d) (3 pt.) In what dimension does the product of capacitance and resistance (i.e. CR) come out?

Time (recall time constant $\tau = RC$ in RC circuit)

(e) (6 pt.) The total magnetic flux through any closed surface should always be zero.

(i) Give a mathematical reason for this. (Just a formula is not acceptable.)

As the magnetic field lines always make closed loops, any field line that enters a surface should always leave $\oint_S \vec{B} \cdot d\vec{A} = 0$



(ii) What is the fundamental physical reason behind this fact?

The underlying physical reason is the fact that there is no magnetic monopole. If we had such monopoles in the universe, this would not be the case.

You can use the space below for another question; please supply necessary routing directions.