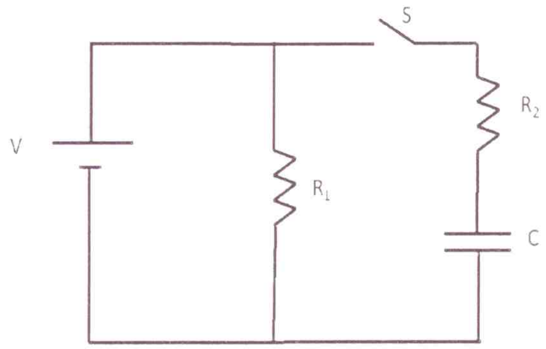
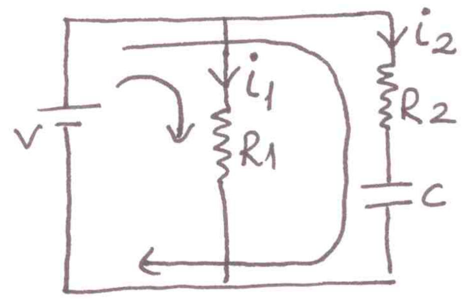
Q.1 *(25 points)* **RC Circuit**

In the circuit shown below the capacitor is <u>initially uncharged</u>. The switch S is closed at t=0. Determine <u>expressions</u> for the currents as a function of time *through R_1 and R_2 and C. In your answers, use <u>only</u> the variables supplied with this problem.*



After the switch is closed $\Rightarrow$



There will be two loops as indicated by arrows in the figure.

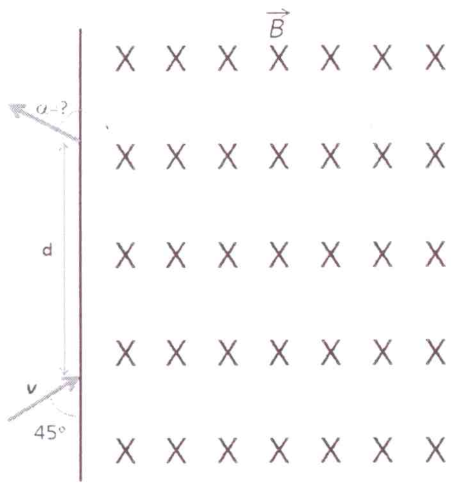
<u>1st Loop:</u> $V - i_1 R_1 = 0 \qquad \underline{\underline{i_1 = \frac{V}{R_1}}}$

<u>2nd Loop:</u> $V - i_2 R_2 - V_C = 0\,; \quad i_2 = \frac{dQ}{dt} \quad ; \quad V_C = \frac{Q}{C}$

$Q = VC\left(1 - e^{-t/R_2C}\right) \qquad i_2 = \frac{dQ}{dt}$

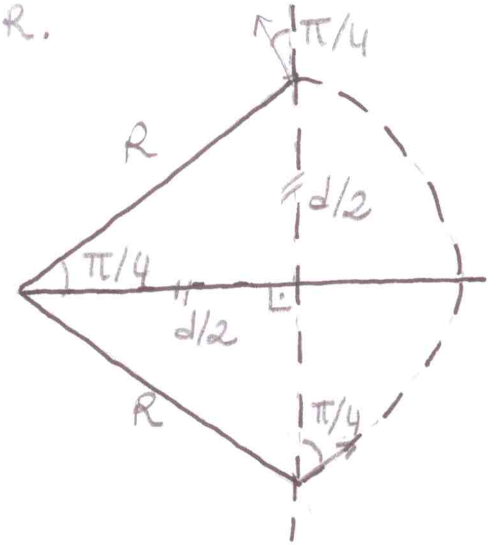
$\frac{dQ}{dt} = V\cancel{C} \cdot \frac{1}{R_2 \cancel{C}} e^{-t/R_2C} \Rightarrow \frac{dQ}{dt} = i_2 = \underline{\underline{\frac{V}{R_2} e^{-t/R_2C}}}$

Q.2 *(25 points)* **Magnetic Field**



A proton moving with speed v=1.3x 10^5 m/s in a field free region abruptly enters uniform magnetic field region with B=0.850 T (**B** $\perp$ **v**). If the proton enters the magnetic field region at a 45° angle as shown in the figure a) at what angle does it leave b) at what distance ***d*** does it exit the field?

Motion of a charged particle under the action of a magnetic field is always motion with constant speed. Particle moves on a circular orbit with a radius R.



$R = \frac{mv}{qB}$;

From the geometry of the path $\alpha = \frac{\pi}{4} = 45°$ &

$R^2 = \frac{d^2}{4} + \frac{d^2}{4} \Rightarrow R = \frac{d}{\sqrt{2}}$;

$d = R\sqrt{2}$, since $R = \frac{mv}{qB}$

$d = \sqrt{2}\,\frac{mv}{qB}$; insert the numeric values $\Rightarrow$

$m_p = 1.67 \times 10^{-27}$ kg

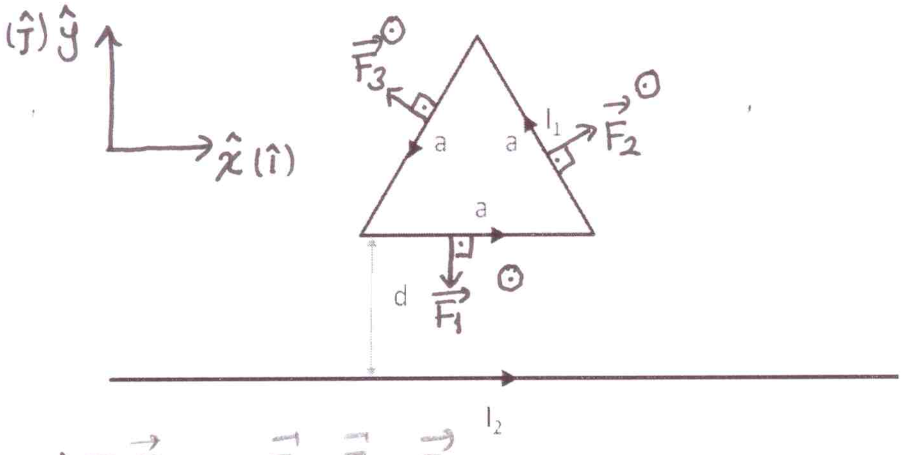
$q = 1.6 \times 10^{-19}$ C

$B = 0.85$ T

$v = 1.3 \times 10^5$ m/s

$d = 1.1$ mm

Q.3 *(25 points)* Magnetic Force

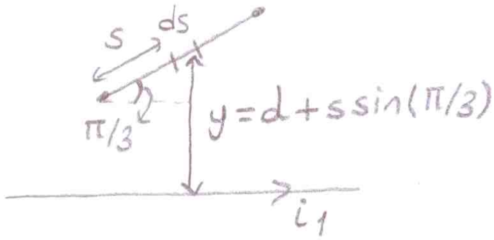


A triangular loop of side length ***a*** carries a current I_1. If the loop is placed a distance ***d*** away from a very long straight wire carrying a current I_2 , a)determine the force on the loop and b) determine the force on the straight wire.

a) $\sum \vec{F}_{loop} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$\vec{F_1} = i_1 \vec{L} \times \vec{B} = i_1 a B$; $B = \frac{i_2 \mu_0}{2\pi d}$ $\Rightarrow$ $\vec{F_1} = -\frac{i_1 i_2 \mu_0 a}{2\pi d} \hat{j}$

$|\vec{F_3}| = |\vec{F_2}|$ $\quad d\vec{F} = i_1 \, d\vec{s} \times d\vec{B}$



$|\vec{F_3}| = -|F_3| \cos(\pi/6)\hat{i} + |F_3| \sin(\pi/6)\hat{j}$

$dF = i_1 \, ds \, \frac{i_2 \mu_0}{2\pi(d + s.\sin(\pi/3))}$

$dF = \frac{i_1 i_2 \mu_0}{2\pi} \frac{ds}{d + \frac{\sqrt{3}}{2} s}$; $F = \int dF = \int_0^a \frac{ds}{d + \frac{\sqrt{3}}{2} s} \frac{i_1 i_2 \mu_0}{2\pi}$

$F = |\vec{F_3}| = \frac{i_1 i_2 \mu_0}{2\pi} \int_0^a \frac{ds}{d + \frac{\sqrt{3}}{2} s}$; change of variables $\Rightarrow$ let $u = d + \frac{\sqrt{3}}{2} s$

Limits $\Rightarrow$ $s: 0 \to a$ $\quad du = \frac{\sqrt{3}}{2} ds$

$u = d \to d + \frac{\sqrt{3}}{2} a$

$|\vec{F_3}| = \frac{i_1 i_2 \mu_0}{2\pi} \int_d^{d + \frac{\sqrt{3}}{2} a} \frac{2}{\sqrt{3}} \frac{du}{u} =$

$= \frac{i_1 i_2 \mu_0}{\sqrt{3}\pi} \int_d^{d + \frac{\sqrt{3}}{2} a} \frac{du}{u} = \frac{i_1 i_2 \mu_0}{\sqrt{3}} \ln u \Big|_d^{d + \frac{\sqrt{3}}{2} a} = \frac{i_1 i_2 \mu_0}{\sqrt{3}\pi} \ln \frac{d + \frac{\sqrt{3}}{2} a}{d} =$

$|\vec{F_3}| = \frac{i_1 i_2 \mu_0}{\sqrt{3}\pi} \ln\left(1 + \frac{\sqrt{3} a}{2d}\right)$; $\sum \vec{F}_{loop} = \vec{F_1} + \vec{F_2} + \vec{F_3}$; $\vec{F}_{2\hat{i}} = -\vec{F}_{3\hat{i}}$

$\sum \vec{F}_{loop} = (-|\vec{F_1}| + 2|\vec{F_3}| \sin(\pi/6))\hat{j}$; $\sum \vec{F} = \left[\frac{i_1 i_2 \mu_0}{\sqrt{3}\pi} \ln\left(1 + \frac{\sqrt{3} a}{2d}\right) - \frac{i_1 i_2 \mu_0 a}{2\pi d}\right] \hat{j}$

b) $F_{loop} = +\sum \vec{F} = -\vec{F}_{wire}$

Q.4 *(25 points)* Conceptual Questions

Answer the following questions **in** the box for each part. To claim any credit for your discussions, **base your reasoning on physics (laws).**

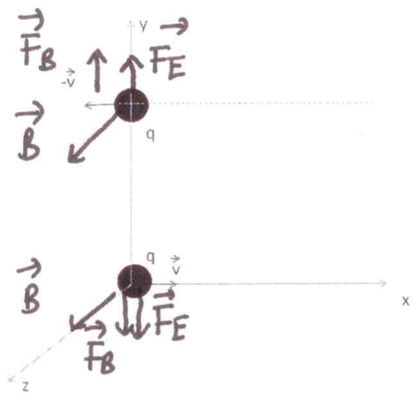
(a) (3 pt.) What is the fundamental conservation law behind Kirchhoff's Junction Rule?

Conservation of electric charge

(3 pt.) What is the fundamental conservation law behind Kirchhoff's Loop Rule?

Conservation of electrostatic energy

(b) (6 pt.) Two protons move parallel to X axis in opposite directions at the same speed. At the instant shown in the figure, show the direction of the electric and magnetic forces between the protons.

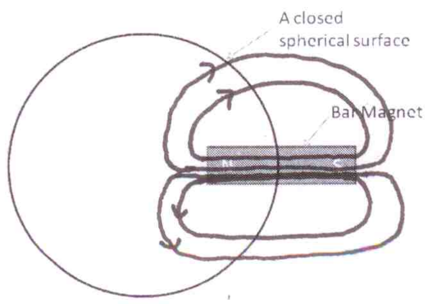


(c) (6 pt.) Calculate the resistance and peak current in a 1000 W hair dryer connected to 220 V line.

$P = iV \; ; \; i = \dfrac{1000\,W}{220\,V} = 4.5\,A$

$V = iR \; ; \; R = \dfrac{V}{i} = 49\,\Omega$

(d) (7 pt.) The figure shows a closed surface enclosing half of a bar magnet. Draw the magnetic field lines and estimate the total magnetic flux through the closed surface. Why?



Magnetic field lines make closed loops.

$\Phi = 0$; there is no magnetic monopole!