PHYS 102 – General Physics-II, Final Exam

Duration: 120 minutes 23 May 2012

NAME:________________________________ **Section:**_______

Q.1 (25)	Q.2 (25)	Q.3 (25)	Q.4 (25)	Total (100)

You must sign the Honor Code for your exam to be graded:

"I pledge, on my Honor, not to lie, cheat, or steal in either my academic or personal life. I understand that such acts violate the Honor Code and undermine the community of trust of which we are all stewards."

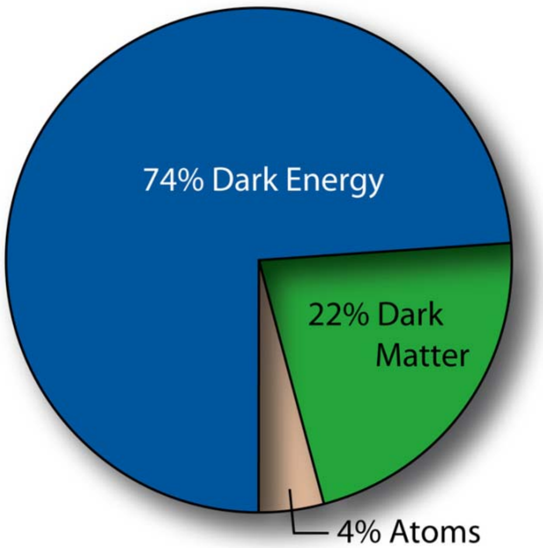
I agree to abide by this Honor Code during this exam.	**Signature:**

Don't forget to sign!

Present your work in a legible and organized format, otherwise you may lose significant portion of your credit even if your solution is correct!

Physics Wiki (No relevance to the exam)

Current physics explains atoms, stars, planets, etc. The biggest recent shock was that they only make up 4% of the Universe. There also exists the so-called **Dark Matter** comprising 22% of the Universe that has only been detected indirectly by its gravity. This matter, different from atoms etc, does not emit or absorb light. Even worse, 74% of the universe, is composed of **Dark Energy**, that acts as a sort of *anti-gravity*. This energy, distinct from dark matter, is responsible for the present-day acceleration of the universal expansion. Yet another enigma is the absence of symmetry between matter and antimatter in the observed Universe. Your physics Textbook will change tremendously within this century!



Composition of the Universe Source: www.nasa.org

Solutions will be posted to → http://www.fen.bilkent.edu.tr/~phys102/

Some expressions from the Textbook which may or may NOT be useful for this exam

(However, you are supposed to know what each symbol means)

Coulomb's Law & Electric Field:

$F = \frac{1}{4\pi\varepsilon_0}\frac{\lvert q_1 q_2\rvert}{r^2}$	$\vec{E} = \frac{\vec{F}_0}{q_0}$	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$	(For a point charge, q)

Torque and Potential Energy of an Electric Dipole in an Electric Field:

$\vec{\tau} = \vec{p} \times \vec{E}$	$U = -\vec{p}\cdot\vec{E}$

Gauss's Law: $\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$	$E_\perp = \frac{\sigma}{\varepsilon_0}$ (Perpendicular component of the electric field at the surface of a conductor)

Electric Potential & Potential Energy:

$V = \frac{U}{q_0}$	$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$ (Collection of point charges)	$U = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$ (q_0 with other point charges)

$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$ (For a charge distribution)	$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$	$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$

Capacitance:

$C = \frac{Q}{V_{ab}}$	$C = \varepsilon_0\frac{A}{d}$ (Parallel plate)	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$ (series)	$C_{eq} = C_1 + C_2 + \cdots$ (parallel)

Energy in a Capacitor:

$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$	$u = \frac{1}{2}\varepsilon_0 E^2$ (Stored energy density)

Capacitors with Dielectrics:

$C = KC_0$	$C = K\varepsilon_0\frac{A}{d} = \varepsilon\frac{A}{d}$ (Parallel plate)	$u = \frac{1}{2}K\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2$	$\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\varepsilon_0}$

Current and current density: $I = \frac{dQ}{dt} = n\lvert q\rvert v_d A,\qquad \vec{J} = nq\vec{v}_d$	**Resistivity and Ohm's Law:** $\rho = \frac{E}{J},\qquad V = IR$

Kirchhoff's Rules: $\sum I = 0$ (junction rule), $\sum V = 0$ (loop rule)	**Power:** $P = V_{ab}I$

Equivalent Resistance:

$R_{eq} = R_1 + R_2 + \cdots + R_n$ (in series)	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$ (in parallel)

Simple R-C Circuit:

Charging:	Discharging:
$q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right) = Q_f\left(1 - e^{-t/(RC)}\right)$ $i(t) = \dfrac{dq}{dt} = \dfrac{\mathcal{E}}{R} e^{-t/(RC)} = I_0 e^{-t/(RC)}$	$q(t) = Q_0 e^{-t/(RC)}$ $i(t) = \dfrac{dq}{dt} = -\dfrac{Q_0}{RC} e^{-t/(RC)}$

Magnetic Forces:

$\vec{F}_m = q\vec{\upsilon} \times \vec{B}$ (on a point charge q)	$d\vec{F}_m = I\, d\vec{l} \times \vec{B}$ (on a current-carrying segment $d\vec{l}$)

Magnetic Flux:	Magnetic Torque and Potential Energy:
$\Phi_B = \int \vec{B} \cdot d\vec{A}, \quad \oint \vec{B} \cdot d\vec{A} = 0$ (for a closed surface)	$\vec{\tau} = \vec{\mu} \times \vec{B}, \quad \vec{\mu} = I\vec{A}, \quad U = -\vec{\mu} \cdot \vec{B}$

Hall Effect:	Cyclotron Radius:
$nq = \dfrac{-J_x B_y}{E_z}$	$R = \dfrac{m\upsilon}{\lvert q \rvert B}$

Magnetic Field of a moving charge *q*:	Magnetic Field of a Current-Carrying Segment:
$\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{q\vec{\upsilon} \times \hat{r}}{r^2}$	$d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{Id\vec{l} \times \hat{r}}{r^2}$

Magnetic Field of a Long Straight Wire:	Magnetic Force Between Two Parallel Wires:
$B = \dfrac{\mu_0 I}{2\pi r}$	$\dfrac{F}{L} = \dfrac{\mu_0 I I'}{2\pi r}$

Axial Magnetic Field of a Current Loop of Radius *a*:	Ampere's Law:
$B_x = \dfrac{\mu_0 N I}{2a}$ (at the center of *N* circular loops)	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

Faraday's Law, Motional emf, Induced Electric Fields:

$\mathcal{E} = -\dfrac{d\Phi_B}{dt}$	$\mathcal{E} = \upsilon BL, \quad \mathcal{E} = \oint \left(\vec{\upsilon} \times \vec{B}\right) \cdot d\vec{l}$	$\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$

Displacement Current & Maxwell's Equations:

$i_D = \varepsilon \dfrac{d\Phi_E}{dt}$	$\oint \vec{B} \cdot d\vec{A} = 0$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \varepsilon_0 \dfrac{d\Phi_E}{dt} \right)_{encl}$

Inductance:

$\mathcal{E} = -L\dfrac{di}{dt}, \; L = \dfrac{N\Phi_B}{i}$	$U = \dfrac{1}{2} L I^2, \; u = \dfrac{B^2}{2\mu}$	$\tau = \dfrac{L}{R}$	$\omega = \sqrt{\dfrac{1}{LC}}$

Electromagnetic Waves:

$E = cB, \; c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$	$\vec{S} = \dfrac{1}{\mu_0} \vec{E} \times \vec{B}$	$I = S_{av} = \dfrac{E_{\max} B_{\max}}{2\mu_0}$	$cu = S$	$p_{rad} = I / c$ (total absorber) $p_{rad} = 2I / c$ (perfect reflector)

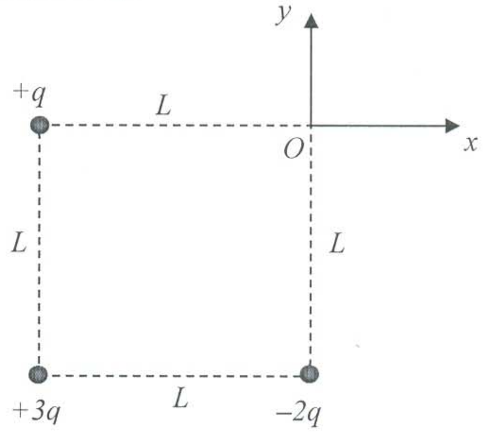
Q.1 Electrostatics (25 points)

Three different point charges are arranged at the corners of a square of side L shown in the figure.

a) What is the potential at the fourth corner (taken as the origin, point O)?

b) What is the electric field <u>vector</u> at point O?

Do not forget the unit vectors, simplify your expressions



a)

$$V(0) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{L} - \frac{2}{L} + \frac{3}{\sqrt{2}L}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 L}\left(\frac{3\sqrt{2}-2}{2}\right)$$

b)

$$\vec{E}(0) = \frac{1}{4\pi\varepsilon_0}\left[q\frac{\hat{i}}{L^2} - 2q\frac{\hat{j}}{L^2} + 3q\frac{(\hat{i}+\hat{j})/\sqrt{2}}{2L^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 L^2}\left[\hat{i}\,\frac{4+3\sqrt{2}}{4} - \hat{j}\,\frac{8-3\sqrt{2}}{4}\right]$$

For part (**a**), put your final answer in this box

$$\frac{q}{4\pi\varepsilon_0 L}\left(\frac{3\sqrt{2}-2}{2}\right)$$

For part (**b**), put your final answer in this box

$$\frac{q}{4\pi\varepsilon_0 L^2}\left[\hat{i}\,\frac{4+3\sqrt{2}}{4} - \hat{j}\,\frac{8-3\sqrt{2}}{4}\right]$$

Q.2 Electromagnetic Induction (25 points)

A slidewire generator is shown in the figure where there is a perpendicular constant magnetic field *B*, uniform everywhere. Assume that the total resistance of the rectangular loop to be only due to the sliding part given by *R*. Now, if the sliding rod (length *L*) is pulled at a constant velocity *v* as shown in the figure, find

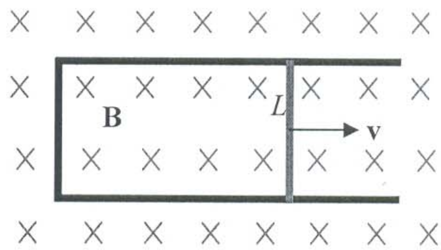
a) The rate at which energy is dissipated around this loop

b) The rate at which mechanical work is done to move the rod through the magnetic field

The dimensions of the stationary parts of the loop are not specified.

In your final results, use only variables provided in problem statement.

This is the solved example 29.6 from our Textbook



First find the induced emf:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{Lv\,dt}{dt} = -BLv$$

a) $P_{dissipated} = I^2R = \frac{\mathcal{E}^2}{R} = \frac{B^2L^2v^2}{R}$

b) As soon as the current I starts flowing around the loop, a force is produced as $F = ILB = \frac{BLv}{R} \cdot LB = \frac{B^2L^2v}{R}$

Mechanical power is $P_{mech} = Fv = \frac{B^2L^2v^2}{R}$

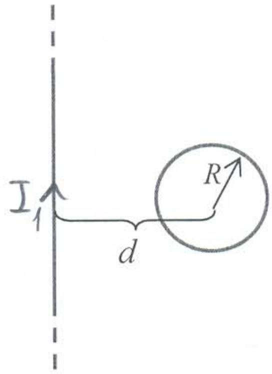
Note that $P_{dissipated} = P_{Mech}$ ... conservation of energy!

For part (a), put your final expression in this box	For part (b), put your answer in this box
$\frac{B^2L^2v^2}{R}$	same as (a)

Q.3 Mutual Inductance (25 points)

Determine the magnitude of the mutual inductance between an infinite straight wire, and a circular loop of radius R, that has its center a distance d from the axis of the straight wire ($d>R$).

Hint: $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$



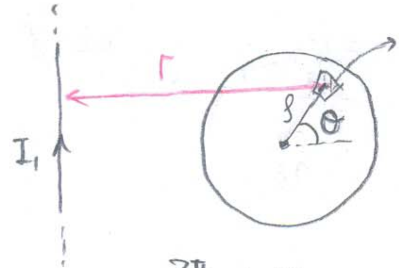
Let's apply a current I on the straight wire and find out the flux passing through the loop.

From Ampere's law $B_1 = \frac{\mu_0 I_1}{2\pi r}$

$$\Phi_{21} = \int B_1 \cdot \hat{n}\, da = \frac{\mu_0 I_1}{2\pi} \int_0^R \rho\, d\rho \int_0^{2\pi} \frac{d\theta}{r}$$

$d + \rho\cos\theta$

$da = \rho\, d\rho\, d\theta$



Using the hint: $\int_0^{2\pi} \frac{d\theta}{d+\rho\cos\theta} = \frac{2\pi}{\sqrt{d^2-\rho^2}}$, $\Phi_{21} = \mu_0 I_1 \int_0^R \frac{\rho\, d\rho}{\sqrt{d^2-\rho^2}}$

Let $u \equiv d^2 - \rho^2$, $\rho\, d\rho = -\frac{du}{2}$; $-\frac{1}{2}\int \frac{du}{\sqrt{u}} = d - \sqrt{d^2-R^2}$

$$\Rightarrow M_{21} = \frac{\Phi_{21}}{I_1} = \mu_0 \left(d - \sqrt{d^2-R^2}\right)$$

Put your final answer in this box

$$\mu_0 \left(d - \sqrt{d^2-R^2}\right)$$

Q.4 Electromagnetic Waves (25 points)

Helium-Neon lasers are often used in physics demonstrations. Let the laser light have a wavelength of λ and a power P that is uniformly spread over a cylindrical beam of diameter d. The medium is to be taken as free-space.

(a) What is the intensity of this laser beam?

(b) What are the maximum values of the electric and magnetic fields?

(c) What is the average energy density in the laser beam?

Give your answers in terms of fundamental constants and variables provided in problem.

This is essentially problem 32.29 from our Textbook

a) $I = \frac{P}{A} = \frac{P}{\pi (d/2)^2}$

A ← area of the laser beam

b) Since $E = cB$, $E_{max} = c B_{max}$

Using $I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2c\mu_0} = \frac{c B_{max}^2}{2\mu_0}$

$$\Rightarrow E_{max} = \sqrt{2c\mu_0 I} = \left[2 c\mu_0 \frac{P}{\pi (d/2)^2}\right]^{1/2} = \left[2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{P}{\pi (d/2)^2}\right]^{1/2}$$

$$B_{max} = \sqrt{2 \frac{\mu_0}{c} I} = \left[2 \frac{\mu_0}{c} \frac{P}{\pi (d/2)^2}\right]^{1/2}$$

c) Since $u = \frac{S}{c}$, $u_{av} = \frac{S_{av}}{c} = \frac{P}{c\pi (d/2)^2}$

For part (**b**), put your final answers in this box

$E_{max} = \sqrt{\frac{8 P c \mu_0}{\pi d^2}}$

$B_{max} = \sqrt{\frac{8 P \mu_0}{c \pi d^2}}$

For part (**a**), put your final answer in this box

$\frac{4P}{\pi d^2}$

For part (**c**), put your final answer in this box

$\frac{4P}{c \pi d^2}$