PHYS 102 – General Physics-II, Midterm-I

Duration: 100 minutes 17 March 2012

NAME:.. **Section**:......................

Q.1 (25)	Q.2 (25)	Q.3 (25)	Q.4 (25)	Total (100)

You must sign the Honor Code for your exam to be graded:

"I pledge, on my Honor, not to lie, cheat, or steal in either my academic or personal life. I understand that such acts violate the Honor Code and undermine the community of trust of which we are all stewards."

I agree to abide by this Honor Code during this exam.	**Signature:**

Don't forget to sign!

Present your work in a legible and organized format, otherwise you may lose significant portion of your credit even if your solution is correct!

Physics Wiki (No relevance to the exam)

A **regenerative brake** is an energy recovery mechanism which slows an object down by converting its kinetic energy into another form. This contrasts with conventional braking systems, where the excess kinetic energy is converted to heat by friction in the brake linings and therefore wasted. Underlying principle is Faraday's law of induction. The generated electricity is stored in a battery for later use as in hybrid electric vehicles (HEV). An HEV combines a conventional internal combustion engine propulsion system with an electric propulsion system, thereby produces less emissions than a comparably-sized gasoline car.

A plug-in hybrid car Source: www.bmw.com

Solutions will be posted to → http://www.fen.bilkent.edu.tr/~phys102/

Some expressions from the Textbook which may or may NOT be useful for this exam

(However, you are supposed to know what each symbol means)

Coulomb's Law:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

Electric Field:

$\vec{E} = \dfrac{\vec{F}_0}{q_0}$, $\quad\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}$ (For a point charge, q)

Torque and Potential Energy of an Electric Dipole in an Electric Field:

$\vec{\tau} = \vec{p}\times\vec{E}$, $\quad U = -\vec{p}\cdot\vec{E}$

Gauss's Law:

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$$

$E_\perp = \dfrac{\sigma}{\varepsilon_0}$ (Perpendicular component of the electric field at the surface of a conductor)

Electric Potential Energy:

$U = \dfrac{q_0}{4\pi\varepsilon_0}\sum_i \dfrac{q_i}{r_i}$ (For q_0 in the presence of other point charges)

Electric Potential:

$V = \dfrac{U}{q_0}$, $\quad V = \dfrac{1}{4\pi\varepsilon_0}\sum_i \dfrac{q_i}{r_i}$ (Due to a collection of point charges)

$V = \dfrac{1}{4\pi\varepsilon_0}\int \dfrac{dq}{r}$ (For a charge distribution)

$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$, $\quad \vec{E} = -\left(\hat{i}\dfrac{\partial V}{\partial x} + \hat{j}\dfrac{\partial V}{\partial y} + \hat{k}\dfrac{\partial V}{\partial z}\right)$

Capacitance:

$C = \dfrac{Q}{V_{ab}}$; $\quad C = \varepsilon_0\dfrac{A}{d}$ (For an ideal parallel plate capacitor)

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots$ (In series); $\quad C_{eq} = C_1 + C_2 + C_3 + \cdots$ (In parallel)

Energy in a Capacitor:

$U = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2 = \dfrac{1}{2}QV$; $\quad u = \dfrac{1}{2}\varepsilon_0 E^2$ (Stored energy density)

Capacitors with Dielectrics:

$C = KC_0$; $\quad C = K\varepsilon_0\dfrac{A}{d} = \varepsilon\dfrac{A}{d}$ (For a parallel plate capacitor)

$u = \dfrac{1}{2}K\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon E^2$, $\quad \oint K\vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\varepsilon_0}$

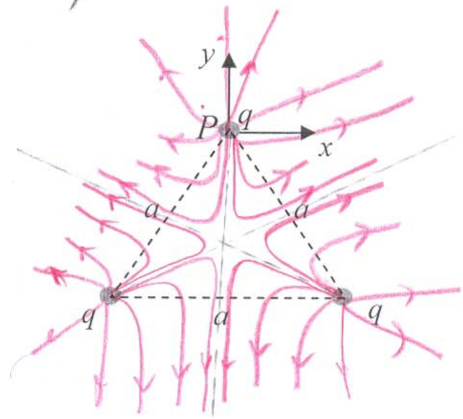
Q.1 Electric Force & Field Lines (25 points)
Three equal positive charges q are at the corners of an equilateral triangle of side a as shown in the figure.

(a) Determine the total electric field vector at point P (at the apex of the triangle), due to two charges at the base of the triangle. (i.e., ignore the charge at P for this part.)

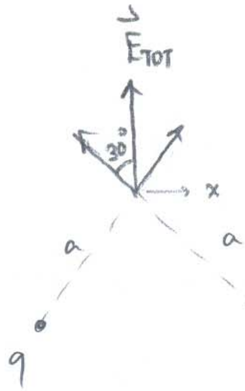
(b) Draw (on the figure) the total electric field lines containing the superposition of all three charges. *(Unclear sketches will lose credit!)*

$$\sin 30^0 = \frac{1}{2},\quad \cos 30^0 = \frac{\sqrt{3}}{2},\quad \sin 60^0 = \frac{\sqrt{3}}{2},\quad \cos 60^0 = \frac{1}{2}$$

b)



a)



The contributions along x dir. will cancel each other.

$$\vec{E}(P) = 2\hat{j}\,\frac{q}{4\pi\varepsilon_0 a^2}\cos 30^\circ$$

$$= \hat{j}\,\frac{\sqrt{3}\,q}{4\pi\varepsilon_0 a^2}$$

For part **(a)**, put your final answer in this box

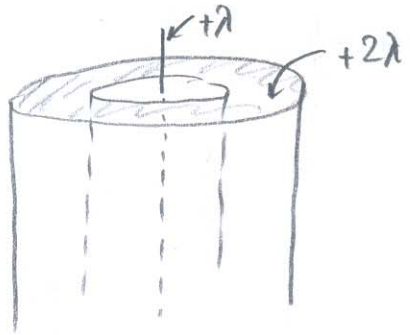
$$\boxed{\hat{j}\,\frac{\sqrt{3}\,q}{4\pi\varepsilon_0 a^2}}$$

Q.2 Gauss's Law (25 points)

A long, straight wire is surrounded by a hollow metal cylinder [with inner radius a and outer radius b] whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. Find,

a) the charge per unit length on the <u>inner</u> surface of the cylinder ($r=a$),
b) the charge per unit length on the <u>outer</u> surface of the cylinder ($r=b$),
c) the electric field magnitude in all three regions: $r<a$, $a<r<b$, and $r>b$.

a)



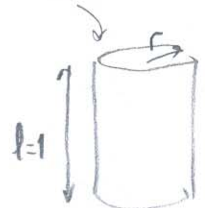
The electric field <u>inside</u> the conductor should vanish. Therefore, on the inner surface there has to be opposite charge to cancel that of the wire (Gauss' law) $\Rightarrow -\lambda$

b) Since the overall linear charge density is $+2\lambda$, the outer surface must have 3λ (so that $-\lambda + 3\lambda = 2\lambda$)

c)

$$\oint_S \vec{E}\cdot\hat{n}\,da = \frac{Q_{enc}}{\varepsilon_0}$$

As E-field lines are along radial dir., choose a cylinder of unit length as a Gaussian surface



$$E \cdot 2\pi r = \frac{Q_{enc.}}{\varepsilon_0} \quad ; \quad Q_{enc} = \begin{cases} +\lambda, & r<a \\ 0, & a<r<b \\ +3\lambda, & r>b \end{cases}$$

For part (**a**), put your final expression in this box

$-\lambda$

For part (**b**), put your final expression in this box

$+3\lambda$

For part (**c**), put your answer in this box

$r<a$ $\quad E(r<a) = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$a<r<b$ $\quad E(a<r<b) = 0$

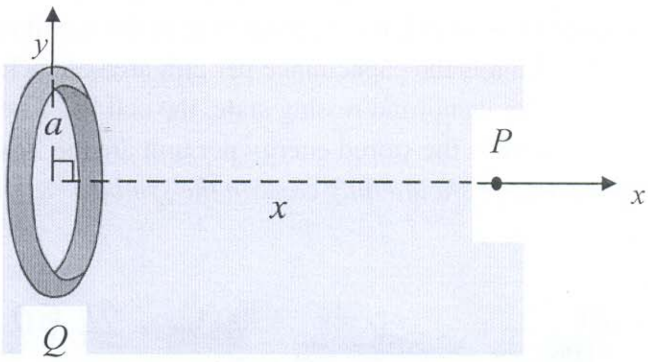
$r>b$ $\quad E(r>b) = \dfrac{3\lambda}{2\pi\varepsilon_0 r}$

Q. 3 Electric Potential and Field (25 points)

Electric charge Q is distributed uniformly around a thin ring of radius a (as shown in the figure).

a) Find the electric potential at a point P on the ring axis at a distance x from the center of the ring,

b) Using this expression in part (a), determine the electric field vector at the same point.



This is solved example 23.14 in the Textbook

a) $$V(x) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{(a^2+x^2)^{1/2}}$$

b) $$\vec{E}(\vec{r}) = -\vec{\nabla} V = -\hat{i}\frac{\partial V}{\partial x}$$

$$= \hat{i} \frac{Q}{4\pi\varepsilon_0} \frac{x}{(a^2+x^2)^{3/2}}$$

For part (a), put your final answer in this box	For part (b), put your final answer in this box
$\frac{Q}{4\pi\varepsilon_0} \frac{1}{(a^2+x^2)^{1/2}}$	$\hat{i} \frac{Q}{4\pi\varepsilon_0} \frac{x}{(a^2+x^2)^{3/2}}$

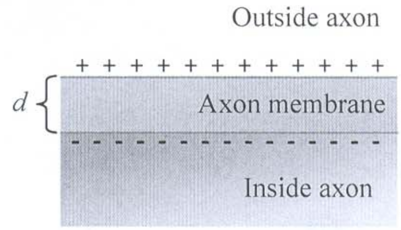
Q. 4 Capacitance (25 points)

Cell membranes (the walled enclosures around a cell) have a thickness to be denoted by d. They are partially permeable to allow charged material to pass in and out, as needed. Equal but opposite charge densities build up on the inside and outside faces of such a membrane, and these charges prevent additional charges from passing through the cell wall. We can model a cell membrane as an ideal parallel plate capacitor, with the membrane itself containing proteins embedded in an organic material to give the membrane a dielectric constant K. (See the figure.)

a) What is the capacitance per unit area of such a cell wall?

b) If in its normal resting state, the cell has a potential difference of V_0 across its membrane, what is the stored energy per unit area in this membrane?

Your answers should only contain the symbols: K, d, V_0, and ε_0



This is essentially Problem 24.50 from the Textbook

a) Using the //-plate expression

$$\bar{C} = \frac{K\varepsilon_0}{d}$$

Capacitance per unit area

b) $$\bar{U} = \frac{1}{2}\bar{C}V_0^2 = \frac{K\varepsilon_0}{2d}V_0^2$$

energy per unit area

For part **(a)**, put your final answer in this box

$$\frac{K\varepsilon_0}{d}$$

For part **(b)**, put your final answer in this box

$$\frac{K\varepsilon_0}{2d}V_0^2$$