PHYS 102 – General Physics-II, Midterm-II

<u>Duration: 100 minutes</u> 21 April 2012

NAME:.. **Section**:......................

Q.1 (25)	Q.2 (25)	Q.3 (25)	Q.4 (25)	Total (100)

You must sign the <u>Honor Code</u> for your exam to be graded:

"I pledge, on my Honor, not to lie, cheat, or steal in either my academic or personal life. I understand that such acts violate the Honor Code and undermine the community of trust of which we are all stewards."

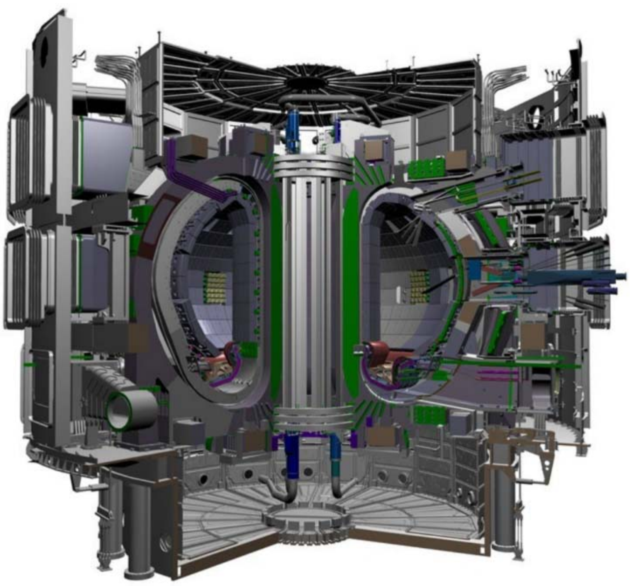
I agree to abide by this Honor Code during this exam.	**Signature:**

Don't forget to sign!

Present your work in a <u>legible</u> and <u>organized</u> format, otherwise you may lose significant portion of your credit even if your solution is correct!

Physics Wiki (No relevance to the exam)

International Thermonuclear Experimental Reactor (ITER) is an international **nuclear fusion** research and engineering project, which is under construction in France, to be completed in 2019. ITER is based on the 'tokamak' concept of magnetic confinement, in which the plasma is contained in a doughnut-shaped vacuum vessel. The fuel—a mixture of deuterium and tritium, two isotopes of hydrogen—is heated to temperatures in excess of 150 million°C, forming a hot plasma. Strong magnetic fields are used to keep the plasma away from the walls. It aims to produce 500 MW of output power for 50 MW of input power, but more importantly, to develop technologies and processes needed for a fusion power plant.



Tokamak Source: www.iter.org

Solutions will be posted to → http://www.fen.bilkent.edu.tr/~phys102/

Some expressions from the Textbook which may or may NOT be useful for this exam

(However, you are supposed to know what each symbol means)

Current and current density:	Resistivity and Ohm's Law:
$I = \frac{dQ}{dt} = n\lvert q\rvert \upsilon_d A, \qquad \vec{J} = nq\vec{\upsilon}_d$	$\rho = \frac{E}{J}, \qquad V = IR$

Kirchhoff's Rules:	Power:
$\sum I = 0$ (junction rule), $\sum V = 0$ (loop rule)	$P = V_{ab} I$

Equivalent Resistance:

$R_{eq} = R_1 + R_2 + \cdots + R_n$ (in series)	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$ (in parallel)

Simple R-C Circuit:

Charging:	Discharging:
$q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right) = Q_f\left(1 - e^{-t/(RC)}\right)$ $i(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/(RC)} = I_0 e^{-t/(RC)}$	$q(t) = Q_0 e^{-t/(RC)}$ $i(t) = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/(RC)}$

Magnetic Forces:

$\vec{F}_m = q\vec{\upsilon} \times \vec{B}$ (on a point charge *q*)	$d\vec{F}_m = I\, d\vec{l} \times \vec{B}$ (on a current-carrying segment $d\vec{l}$)

Magnetic Flux:	Magnetic Torque and Potential Energy:
$\Phi_B = \int \vec{B} \cdot d\vec{A}, \qquad \oint \vec{B} \cdot d\vec{A} = 0$ (for a closed surface)	$\vec{\tau} = \vec{\mu} \times \vec{B}, \quad \vec{\mu} = I\vec{A}, \quad U = -\vec{\mu} \cdot \vec{B}$

Hall Effect:	Cyclotron Radius:
$nq = \frac{-J_x B_y}{E_z}$	$R = \frac{m\upsilon}{\lvert q\rvert B}$

Magnetic Field of a moving charge *q*:	Magnetic Field of a Current-Carrying Segment:
$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{\upsilon} \times \hat{r}}{r^2}$	$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$

Magnetic Field of a Long Straight Wire:	Magnetic Force Between Two Parallel Wires:
$B = \frac{\mu_0 I}{2\pi r}$	$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$

Axial Magnetic Field of a Current Loop of Radius *a*:	Ampere's Law:
$B_x = \frac{\mu_0 N I}{2a}$ (at the center of *N* circular loops)	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

Q.1 RC Circuit (25 points)

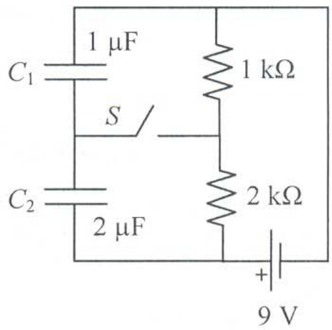
Consider the DC circuit shown in the figure. Determine the final value of the charges in each capacitor separately, for the case when

(a) the switch *S* is open (as in the figure) and a long time has passed

(b) the switch *S* is now closed and a long time has passed in this setting

For both cases, you do not need to know the initial charges on the capacitors.

'μ' means 10^{-6} and 'k' means 10^{3}; give numerical values, do not forget to include units!



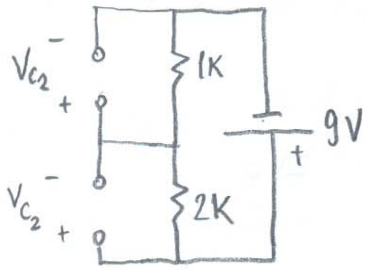
a) When S is open, the resistors have no effect on the capacitors.

The circuit reduces to:

same charge accumulates on both

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{2}{3}\,\mu F \qquad Q = Q(C_1) = Q(C_2) = C_{eq} \cdot V = 6\mu C$$

b) When S is closed and after long time passes in this setting, both capacitors get fully charged. We can replace them with open circuits to find their final voltages V_{C_1}, V_{C_2}:



$$\underbrace{V_{C_1} = \frac{1}{2+1} \cdot 9 = 3V}, \quad V_{C_2} = \frac{2}{2+1} \cdot 9 = 6V$$

$Q_{C_1} = C_1 \cdot V_{C_1}$

$Q_{C_1} = 3\mu C$

$Q_{C_2} = C_2 \cdot V_{C_2}$

$Q_{C_2} = 12\mu C$

For part **(a)**, put your final answers in this box
$Q(C_1) = 6\mu C = 6.10^{-6} C$
$Q(C_2) = 6\mu C = 6.10^{-6} C$

For part **(b)**, put your final answers in this box
$Q(C_1) = 3\mu C = 3.10^{-6} C$
$Q(C_2) = 12\mu C = 12.10^{-6} C$

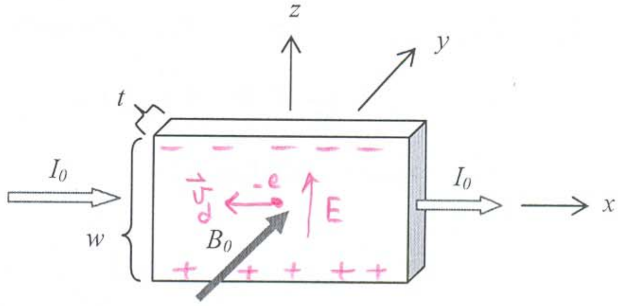
Q.2 Electric & Magnetic Forces (25 points)

You place a metallic sample with thickness t, and width w, in a uniform magnetic field $\hat{j}B_0$ as shown in the figure. When you run a current I_0 in the $+x$ direction, you find that the potential at the bottom of the slab $(z=-w/2)$ is ΔV higher than at the top $(z=+w/2)$.

a) From this measurement, determine the concentration of mobile electrons in the sample. You can denote electron's charge with $-e$.
In your final result, use only variables provided in problem statement.

b) What is the scientific name for this measurement?

This is essentially the solved example 27.12 in our Textbook.



a) The current density:

$$J_x = \frac{I_0}{A} = \frac{I_0}{wt} = n e v_d$$

if you do not want to derive the result

Also from the voltage difference we can find the electric field as:

$$E_z = \frac{\Delta V}{w}$$

Equating the electric and magnetic $\quad -eE_z + e v_d B_y = 0 \Rightarrow E_z = -v_d B_y$

OR directly use the supplied expression on the info sheet: $q \to -e$

$$-ne = \frac{-J_x B_y}{E_z} \Rightarrow \underset{\text{concentration}}{n} = \frac{I_0 B_0}{\underset{\text{thickness}}{t}\, \Delta V e} > 0$$

b) This is the Hall-effect measurement

For part **(a)**, put your final expression in this box

$$n = \frac{I_0 B_0}{t e \Delta V}$$

For part **(b)**, put your answer in this box

Hall Effect

Q.3 Magnetic Field & Magnetic Force (25 points)

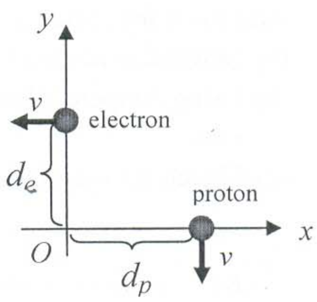
An electron and a proton are each moving at the same speeds *v*, but in perpendicular paths, as shown in the figure. At the instant when they are at the positions shown in the figure, find

a) the total magnetic field (vector) they produce at the origin *O*,

b) the magnetic force (vector) the electron exerts on the proton.

Denote the charge for electron and proton by $-e$ and $+e$, respectively.

This is the simplified version of Ex. 28.8 from our Textbook



a) Make use of $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$

$$\Rightarrow \vec{B}_{tot}(0) = \frac{\mu_0 e v}{4\pi}\left[\underbrace{\frac{-(-\hat{i})\times(-\hat{j})}{d_e^2}}_{e^-} + \underbrace{\frac{-\hat{j}\times(-\hat{i})}{d_p^2}}_{p^+}\right]$$

$$\vec{B}_{tot}(0) = -\hat{k}\,\frac{\mu_0 e v}{4\pi}\,\frac{d_e^2+d_p^2}{d_e^2\cdot d_p^2}$$

b) $$\vec{B}_e(\text{at } p^+ \text{ due to } e^-) = \frac{\mu_0 e v}{4\pi}\left[\frac{-(-\hat{i})\times\overbrace{(-d_e\hat{j}+d_p\hat{i})}^{\vec{R}}}{\underbrace{(d_e^2+d_p^2)^{3/2}}_{R^3}}\right] = -\hat{k}\,\frac{\mu_0 e v}{4\pi}\,\frac{d_e}{(d_e^2+d_p^2)^{3/2}}$$

$$\vec{F}_M(\text{on } p^+ \text{ due to } e^-) = e(-v\hat{j})\times\vec{B}_e$$

$$= \hat{i}\,\frac{\mu_0 e^2 v^2}{4\pi}\,\frac{d_e}{(d_e^2+d_p^2)^{3/2}}$$

For part (**a**), put your final answer in this box	For part (**b**), put your final answer in this box
$-\hat{k}\,\frac{\mu_0 e v}{4\pi}\,\frac{d_e^2+d_p^2}{d_e^2\cdot d_p^2}$	$\hat{i}\,\frac{\mu_0 e^2 v^2}{4\pi}\,\frac{d_e}{(d_e^2+d_p^2)^{3/2}}$

Q.4 Ampere's Law (25 points)

A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is $\vec{J}$. The current density, although symmetric about the cylinder axis, is not constant and varies according to the relationship

$$\vec{J}=\begin{cases}\hat{k}\left(\dfrac{b}{r}\right)e^{(r-a)/\delta}, & \text{for } r\le a\\[2ex] 0, & \text{for } r>a\end{cases},$$

where a is the radius of the cylinder, while b and δ are known constants.

(a) Determine the total current I_0 passing through the entire cross section.

(b) Using Ampere's law, derive an expression for the magnetic field vector for both $r<a$ and $r>a$.

(c) Sketch the magnitude of the magnetic field as a function of r.

This is essentially Problem 28.82 from our Textbook

a) $$I_0=\int_S \vec{J}\cdot\hat{n}\,da=\int_0^a 2\pi r\,dr\,\frac{b}{r}\,e^{(r-a)/\delta}=2\pi b\delta\, e^{(r-a)/\delta}\Big|_0^a$$

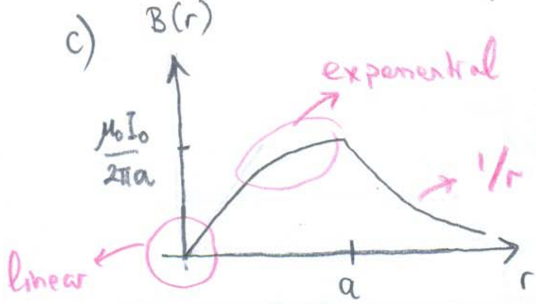
$$\Rightarrow I_0=2\pi b\delta\left[1-e^{-a/\delta}\right]=2\pi b\delta\, e^{-a/\delta}\left[e^{a/\delta}-1\right]$$

b) Ampere's Law: $$\underbrace{\oint_C \vec{B}\cdot d\vec{l}}_{B_\phi 2\pi r}=\mu_0 I_{enc}=\mu_0\begin{cases}2\pi b\delta\, e^{-a/\delta}\left[e^{r/\delta}-1\right], & r\le a\\ I_0, & r>a\end{cases}$$

behaves as 'r' as $\lim_{r\to 0}$

$$\Rightarrow B_\phi=\begin{cases}\mu_0\dfrac{b\delta}{r}e^{-a/\delta}\left[e^{r/\delta}-1\right], & r\le a\\[2ex] \dfrac{\mu_0 b\delta}{r}e^{-a/\delta}\left[e^{a/\delta}-1\right], & r>a\end{cases}=\begin{cases}\dfrac{\mu_0 I_0}{2\pi r}\,\dfrac{e^{r/\delta}-1}{e^{a/\delta}-1}, & r\le a\\[2ex] \dfrac{\mu_0 I_0}{2\pi r}, & r>a\end{cases}$$

c)



For part **(a)**, put your final answer in this box

$$2\pi b\delta\left[1-e^{-a/\delta}\right]$$

For part **(b)**, put your final expression in this box

$$\frac{\mu_0 b\delta}{r}e^{-a/\delta}\left[e^{r/\delta}-1\right],\quad r\le a$$

$$\frac{\mu_0 b\delta}{r}e^{-a/\delta}\left[e^{a/\delta}-1\right],\quad r>a$$